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A DSGE MODEL FOR LUXEMBOURG
WITHIN THE EURO AREA AND GLOBAL ECONOMY

ALBAN MOURA   KYRIACOS LAMBRIAS

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LU-EAGLE: A DSGE MODEL FOR LUXEMBOURG WITHIN THE EURO AREA AND GLOBAL ECONOMY

ALBAN MOURA AND KYRIACOS LAMBRIAS

Abstract. We describe LU-EAGLE, a DSGE model developed at the Banque centrale du Luxembourg. LU-EAGLE borrows its general structure from the Euro Area and GLobal Economy (EAGLE) model developed by the European System of Central Banks and also embeds specific features to capture some important characteristics of Luxembourg's economy. In particular, the model reproduces the high levels of exports and imports relative to GDP, as well as the significant share of cross-border workers in Luxembourg's labor market. We calibrate LU-EAGLE and discuss simulation results describing the effects of a set of standard shocks, originating both in Luxembourg and abroad. The model suggests that international spillovers make Luxembourg more responsive to monetary policy shocks and less responsive to fiscal policy shocks. Moreover, it highlights how fluctuations in foreign demand have a significant impact on domestic developments.

JEL Codes: C54, E17, E32, E37, E62, F47.

Keywords: DSGE models, open economy models, policy analysis, Luxembourg.

Moura: Banque centrale du Luxembourg (BCL), Département Economie et Recherche, 2 boulevard Royal, L-2983 Luxembourg, Luxembourg (e-mail: alban.moura@bcl.lu). Lambrias: European Central Bank (ECB), Directorate General Economics, Sonnemannstrasse 20, 60314 Frankfurt am Main, Germany (kyriacos.lambrias@ecb.int). For helpful comments and suggestions, we thank Pascal Jacquinot, Luca Marchiori, Olivier Pierrard, Abdelaziz Rouabah, Jean-Pierre Schoder, and especially Paolo Guarda, as well as participants at presentations given at the BCL and the ECB. This paper should not be reported as representing the views of the Banque centrale du Luxembourg, the European Central Bank, or the Eurosystem. The views expressed are those of the authors and may not be shared by other research staff or policymakers at BCL, ECB, or the Eurosystem.

Résumé Non Technique

L'adoption de modèles d'équilibre général dynamiques et stochastiques (modèles DSGE, en abrégé) par les banques centrales et autres institutions publiques s'est développée au cours des dernières 15 années. Cette classe de modèles constitue un cadre particulièrement adapté à l'analyse des politiques économiques, de par leur nature structurelle et leur traitement cohérent des anticipations du secteur privé. En effet et par opposition avec les modèles macro-économétriques traditionnels, les modèles DSGE sont construits en accord avec la théorie microéconomique. En particulier, ils caractérisent la dynamique agrégée de l'économie à partir du comportement de plusieurs types d'agents (ménages, entreprises, autorité fiscale, autorité monétaire), qui ont chacun un objectif propre, font face à différentes contraintes et interagissent ensemble sur les marchés. Selon leur spécification, ces modèles permettent d'étudier des phénomènes de court terme (cycles économiques, politique monétaire) ou de plus long terme (politique fiscale, démographie, progrès technique).

Cet article décrit le modèle LU-EAGLE, pour LUxembourg within the Euro Area and GLobal Economy, un modèle DSGE développé à la Banque centrale du Luxembourg (BCL) pour analyser la dynamique économique à horizons relativement courts. Pour les analyses de plus long terme, par exemple concernant la soutenabilité du système des retraites, la BCL dispose d'un autre modèle DSGE, dénommé LOLA (Pierrard et Sneessens, 2009 ; Marchiori et Pierrard, 2012, 2015). LU-EAGLE et LOLA constituent ainsi des outils complémentaires à disposition de la BCL pour la modélisation et la compréhension des développements économiques au Luxembourg.

LU-EAGLE constitue la version luxembourgeoise du modèle EAGLE (Gomes et al., 2012), développé par le Système Européen des Banques Centrales pour étudier les interactions macroéconomiques entre quatre blocs régionaux : un pays de référence au sein de la zone euro, un agrégat représentant le reste de la zone euro et deux régions hors de la zone euro (les Etats-Unis et le reste du monde). Si le pays de référence était l'Allemagne dans l'article original de Gomes et al. (2012), des banques centrales nationales ont construit d'autres versions du modèle, chacune prenant leur pays comme référence. Naturellement, le Luxembourg est le pays de référence dans LU-EAGLE.

Dans le modèle, les quatre blocs échangent des biens et services, ainsi que des actifs financiers. Les deux blocs membres de la zone euro partagent en outre une autorité monétaire commune. Clairement, la prise en compte de ce type de liens internationaux semble particulièrement important pour analyser la dynamique d'une petite économie ouverte comme le Luxembourg.

Au sein de chaque bloc, LU-EAGLE intègre les éléments habituels des modèles DSGE quantitatifs, tels que des coûts d'ajustement (habitudes de consommation, coûts d'investissement) et des frictions nominales (prix et salaires rigides à court terme). La structure de production est relativement plus complexe que dans les modèles académiques, avec plusieurs types d'entreprises échangeant des biens intermédiaires à la fois au sein de

leur bloc et d'un bloc à l'autre. La prise en compte de la différence entre la production échangeable et la production non échangeable permet aussi d'améliorer la capacité du modèle à expliquer les effets des chocs internationaux, en capturant notamment certains phénomènes de réallocation intersectorielle. Finalement, LU-EAGLE incorpore un secteur fiscal détaillé, ce qui en fait un outil adapté à l'étude des politiques budgétaires.

Par rapport au modèle original de Gomes et al. (2012), LU-EAGLE incorpore également deux éléments additionnels requis pour reproduire certaines caractéristiques de l'économie luxembourgeoise. Premièrement, le Luxembourg étant une (très) petite économie ouverte, le contenu en importations des composantes de la demande finale y est plus élevé que dans d'autres pays. Pour capturer cette propriété et générer un degré d'ouverture comparable à celui mesuré dans les données, LU-EAGLE modélise explicitement les contenus en importations de la consommation privée, de l'investissement, de la dépense publique et des exportations. Deuxièmement, LU-EAGLE prend en compte les travailleurs frontaliers, qui représentent une part significative de la force de travail au Luxembourg, tout en dépensant la majeure partie de leur revenu hors du pays.

Il convient aussi de mentionner que le secteur financier luxembourgeois n'apparait pas explicitement dans le modèle. À la place, il est implicitement inclus dans le secteur des biens et services échangeables (les services financiers représentent environ 50% des exportations du Luxembourg et 45% de ses importations). En général, les modèles DSGE introduisent des banques dans le seul objectif de générer des frictions financières affectant la distribution du crédit; dans ces modèles, les banques n'ont pas d'activité de gestion d'actifs et n'emploient pas de travailleurs. Clairement, un tel cadre ne permettrait pas de capturer la contribution importante du secteur financier à la dynamique économique du Luxembourg. Développer une spécification permettant d'étudier ce type de questions reste un objectif de recherche important.

Après avoir présenté la structure de LU-EAGLE, nous décrivons le processus de calibration du modèle. Chaque fois que possible, les valeurs sont choisies de manière à reproduire certaines régularités empiriques ; le reste des paramètres est calibré en accord avec la littérature sur les modèles DSGE à plusieurs pays. Finalement, afin d'illustrer le fonctionnement de LU-EAGLE et sa capacité à informer les décisions de politique économique, l'article présente une série de simulations décrivant la réponse du modèle à plusieurs chocs standards, survenant au Luxembourg ou à l'étranger. Les résultats suggèrent que le degré d'ouverture de l'économie luxembourgeoise la rend plus sensible aux chocs de politique monétaire et moins sensible aux chocs budgétaires domestiques. Le modèle permet également d'identifier les mécanismes à travers lesquels les fluctuations de la demande externe se propagent à l'économie domestique.

1. Introduction

During the last 15 years, dynamic stochastic general equilibrium (DSGE) models have become an important component of central banks' empirical toolkit.[1] These models provide a useful framework for policy analysis, both because of their structural nature and their coherent treatment of expectations. Indeed, in contrast to traditional macro-econometric models,[2] DSGE models are derived from microeconomic principles: they aim to explain aggregate dynamics from the behavior of different economic agents (households, firms, a government, a central bank…), who all have different objectives and face different constraints, and whose coordination is ensured by market-clearing restrictions in general equilibrium. Depending on their specification, these models can study either short-run (business cycles, monetary policy) or medium- to long-run phenomena (fiscal policy, demography, technological change).

In this paper, we describe LU-EAGLE, a DSGE model developed at the Banque centrale du Luxembourg (BCL). In contrast to the LOLA model already in use at the BCL (Pierrard and Sneessens, 2009; Marchiori and Pierrard, 2012, 2015), which focuses on long-run policy and demographic issues, the structure of LU-EAGLE is more tailored to short-run analysis. The two models thus provide the BCL with complementary tools to understand economic dynamics in Luxembourg.

LU-EAGLE borrows its general structure from the Euro Area and GLobal Economy (EAGLE) model (Gomes, Jacquinot, and Pisani, 2012) developed within the European System of Central Banks (ESCB). Both models study macroeconomic interdependence between four different regions: a reference country within the euro area, an aggregate representing the rest of the euro area, and two regions outside the monetary union (taken to be the United States and the rest of the world). While the reference country was Germany in Gomes, Jacquinot, and Pisani (2012), country-specific versions of EAGLE with different reference countries have been developed by National Central Banks. Naturally, Luxembourg is the reference country in LU-EAGLE. In the models, the four regions trade goods and financial assets, and the euro area members share a common monetary authority. Taking into account these international linkages seems particularly important to analyze economic developments in a small open economy like Luxembourg.

Within each region block, LU-EAGLE features the common elements of quantitative DSGE models (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007), including real frictions (consumption habits, investment adjustment costs) and nominal frictions (sticky prices and wages). It also includes more specific propagation channels, such as a production structure involving several layers of firms trading together within

[1]For instance, both the European Central Bank (New Area Wide Model, Coenen, McAdam, and Straub, 2008) and the Federal Reserve Board (SIGMA model, Erceg, Guerrieri, and Gust, 2006) develop and use DSGE models in their research departments. Other policy-making institutions, such as the International Monetary Fund or the European Commission, also work with DSGE models.

[2]Guarda (2005) presents the traditional macro-econometric model used for forecasting and policy analysis at the Banque centrale du Luxembourg.

and across countries. The distinction between traded and nontraded goods also improves the model's ability to capture the international transmission of foreign and domestic shocks. Finally, its rich fiscal side makes LU-EAGLE particularly attractive to study the effects of alternative budgetary plans.

Compared to the benchmark EAGLE model, LU-EAGLE introduces two additional dimensions that are required to match some key characteristics of Luxembourg's economy. First, given the small size and openness of Luxembourg, both government expenditures and exports feature a high import content. To capture this stylized fact, we follow Brzoza-Brzezina, Jacquinot, and Kolasa (2014) and Jacquinot, Clancy, and Lozej (2014) by allowing the export good and the government good to be produced using both local and foreign inputs. In addition to helping to reproduce some relevant steady-state features, such as the GDP shares of exports and imports (and thus also the trade balance), this mechanism improves the modeling of the transmission of government spending shocks to the economy and leads to more plausible fiscal multipliers. Second, LU-EAGLE takes into account cross-border workers, who represent a significant share of Luxembourg's workforce but spend most of their income outside the country. To introduce these agents in the model, we assume that the labor services supplied by resident and cross-border workers in Luxembourg are sufficiently differentiated, so that domestic firms need both as inputs for production. We also modify some accounting identities, such as the current account in Luxembourg and in the rest of the euro area, to account for the wages paid to cross-border workers.

Finally, we must clarify that Luxembourg's financial industry does not appear as a separate sector in the model. Rather, it is implicitly included in the tradables sector since financial services represent about 50% of Luxembourg's exports and 45% of its imports. DSGE models generally assume that banks are only active in the markets for loans and deposits, where they generate financial frictions. In particular, in such models banks do not manage assets, nor do they hire workers.[3] Clearly, such a framework could not capture the important contribution of the financial sector to Luxembourg's growth or labor market dynamics. Developing an alternative specification to study these questions seems an important task for future research.

We calibrate LU-EAGLE to Luxembourg, the rest of the euro area, the United States, and the rest of the world. Whenever possible, we set model parameters to match empirical regularities in the data, and fix the remaining parameters based on the earlier literature on multi-country DSGE models. We then present a set of simulations to describe macroeconomic dynamics in Luxembourg, as captured by the model, following a set of standard shocks originating both in Luxembourg and abroad. These experiments are useful to illustrate the workings of LU-EAGLE, as well as to highlight how it can support policy advice. Our results suggest that international spillovers make Luxembourg more responsive to monetary policy shocks and less responsive to domestic fiscal policy shocks.

[3]For example, see the version of the EAGLE model with financial linkages (Jacquinot, Pisani, Gomes, Bokan, and Gerali, 2016).

They also highlight how fluctuations in foreign demand may have significant effects on domestic developments.

The rest of the paper is organized as follows. Section 2 provides an overview of the benchmark EAGLE model and describes the additional features introduced in LU-EAGLE. Section 3 explains the calibration of the model. Finally, Section 4 reports the results of the simulation exercises and Section 5 concludes.

2. Model

This section briefly reviews the core characteristics of the original EAGLE model, that are maintained in LU-EAGLE. It then presents the additional elements introduced in LU-EAGLE to provide a better representation of Luxembourg's economy.

2.1. **Benchmark EAGLE model.** EAGLE is a DSGE model of the euro area within the global economy that describes the general equilibrium of a world represented by four regions, two of which belong to the euro area. The four regions are modeled in a symmetric fashion, except for monetary policy which is common within the euro area and region-specific outside. Below, we provide a sketch of the model and refer the interested reader to the paper by Gomes, Jacquinot, and Pisani (2012) for a full description.

Within each region, infinitely-lived households consume final goods, accumulate the capital stock, and supply labor services to the production sector. The model distinguishes between two types of households, depending on their ability to participate in financial markets: unconstrained agents have full access to asset markets and can transfer their wealth over time using money, bonds, and physical capital, whereas constrained agents are excluded from asset markets and can only use money to smooth their intertemporal consumption profile. These households help the model generate Keynesian effects of public expenditure, even though they are more sophisticated than the 'rule-of-thumb' agents sometimes considered in DSGE models.[4] Labor markets are monopolistically competitive, introducing wage rigidity in the model. The set up follows Calvo (1983), so that only a fraction of wage contracts can be reoptimized each period. Contracts that are not reoptimized adjust the wages to a weighted average of past and steady-state consumer inflation.

The production side is composed of several layers. An intermediate sector produces both tradable and nontradable goods from capital and labor services, using Cobb-Douglas technologies. These intermediate firms also set prices following a Calvo scheme, introducing price rigidity in the model. Tradable goods can be sold both domestically and internationally, while nontradable goods serve only the domestic market. Export prices are set in the currency of the destination market, limiting the pass-through of exchange rate fluctuations to import prices.[5] Competitive firms produce the final goods usable

[4]Unlike the constrained households in EAGLE, rule-of-thumb agents do not optimize but simply consume all their current income.

[5]Since Luxembourg trades mostly with the other euro-area countries, which share the same currency, this local currency pricing assumption has limited effects in LU-EAGLE.

for private consumption, public consumption, and investment by aggregating domestic nontradables, domestic tradables, and imported tradables using constant-elasticity-of-substitution (CES) production functions.

While physical capital is domestically owned, unconstrained households from each block participate in international financial markets in which they trade riskless bonds denominated in US dollars.[6] For these agents, an uncovered interest parity condition holds, linking the interest rate differential to the expected change in the exchange rate of the domestic currency against the worldwide core currency (taken to be the US dollar). The steady-state levels of each block's international investment position are calibrated and, in dynamic simulations, these external asset positions respond to variations in current accounts to maintain the worldwide general equilibrium.

Finally, EAGLE is quite developed on the policy side. Each region is endowed with a fiscal authority that raises revenues from taxes levied on the private sector and seigniorage earned from monetary creation. The model features a rich set of tax instruments, that includes a VAT-like tax on consumption expenditures and taxes on labor and capital income. These tax revenues are used to purchase the final government good, entirely produced from domestic nontradables, as well as to finance transfer payments to households. Any resulting public debt is financed by government bonds and convergence toward a long-run debt-to-GDP ratio is ensured by a smooth fiscal rule. Regarding monetary policy, the two regions within the euro area share a common central bank, while the two other regions each have their own monetary authority. All central banks set their target policy interest rate according to a Taylor-type rule reacting to consumer price inflation and the output gap.

2.2. **Specific features of LU-EAGLE.** To better capture macroeconomic dynamics in Luxembourg, we augment the original EAGLE setup with two features. First, we allow imports to be used as inputs in production for government consumption and exports. Second, we modify the labor market structure of the Luxembourg block of the model to account for cross-border workers. We present these changes in the following sections.

2.2.1. *Role of imports.* Small open economies like Luxembourg tend to specialize in a few sectors and to rely on imports to obtain the goods and services that are not produced domestically. As a result, the import content of private consumption, investment, government expenditure, exports, and overall GDP are typically higher than in larger economies. For instance, imports represented on average 165% of Luxembourg's GDP over the period 2010-2016, compared with about 20% for the euro area as a whole.

Because the original EAGLE model assumes that the final goods used for government expenditures and exports are produced using domestic inputs only, it cannot generate appropriate import shares for Luxembourg. Following Brzoza-Brzezina, Jacquinot, and Kolasa (2014) and Jacquinot, Clancy, and Lozej (2014), we solve this issue by making

[6]The two euro-area blocks of the model also trade a riskless euro-denominated bond.

imported intermediate goods a required input in production for government consumption and exports.

Regarding government consumption, we postulate a nested CES function similar to those used for private consumption and investment. In particular, in LU-EAGLE government consumption Q^G is produced according to

$$Q_t^G = \left[\nu_G^{\frac{1}{\mu_G}} (TT_t^G)^{\frac{\mu_G - 1}{\mu_G}} + (1 - \nu_G)^{\frac{1}{\mu_G}} (NT_t^G)^{\frac{\mu_G - 1}{\mu_G}} \right]^{\frac{\mu_G}{\mu_G - 1}}, \tag{1}$$

where NT^G is a basket of domestic nontradable inputs and TT^G is a bundle of domestic and imported tradable inputs. The structural parameters $\nu_G \in [0, 1]$ and $\mu_G > 0$ denote the relative weight of tradable to nontradable inputs and their elasticity of substitution. The tradable bundle is given by

$$TT_t^G = \left[\nu_{TG}^{\frac{1}{\mu_{TG}}} (HT_t^G)^{\frac{\mu_{TG} - 1}{\mu_{TG}}} + (1 - \nu_{TG})^{\frac{1}{\mu_{TG}}} (IM_t^G)^{\frac{\mu_{TG} - 1}{\mu_{TG}}} \right]^{\frac{\mu_{TG}}{\mu_{TG} - 1}}, \tag{2}$$

where HT^G and IM^G denote domestic and imported tradable inputs used in the production of government consumption. As above, $\nu_{TG} \in [0, 1]$ and $\mu_{TG} > 0$ denote the relative weight of domestic tradables to imports and their elasticity of substitution.

Exports are produced using tradable inputs only, from both domestic and foreign sources. Hence, we assume a production function of the form

$$Q_t^X = \left[\nu_{TX}^{\frac{1}{\mu_{TX}}} (HT_t^X)^{\frac{\mu_{TX} - 1}{\mu_{TX}}} + (1 - \nu_{TX})^{\frac{1}{\mu_{TX}}} (IM_t^X)^{\frac{\mu_{TX} - 1}{\mu_{TX}}} \right]^{\frac{\mu_{TX}}{\mu_{TX} - 1}}, \tag{3}$$

where Q^X denotes the quantity of exports, HT^X and IM^X refer to the domestic and imported tradable inputs used in the export sector, and $\nu_{TX} \in [0, 1]$ and $\mu_{TX} > 0$ parametrize the relative weight of domestic inputs to imports and their elasticity of substitution.

Thanks to the CES structure, deriving the various input demand functions and the implied price indexes is straightforward. The other necessary adjustments, for instance to market clearing conditions, are also immediate, so we relegate all this material to Appendix B. Here, we limit discussion to the device used to capture sticky export prices in LU-EAGLE. In particular, since the homogeneous export good in equation (3) is produced by a competitive sector, we need to introduce an additional layer to generate pricing frictions in the export market. We proceed as in Christiano, Trabandt, and Walentin (2011) and assume that the homogeneous export good is purchased by a continuum of domestic export firms, which convert it into specialized varieties in a brand-naming process. These exporters then serve the foreign markets and set export prices in the currency of the destination markets subject to Calvo frictions, as described in the appendix.

2.2.2. *Cross-border workers.* Since the 1990's, the Luxembourg economy has increasingly relied on cross-border workers. Between 1995 and 2017, their share in the total workforce rose from about 20% of jobs to 40%, and they receive a similar share of the aggregate wage bill. This aspect of the national economy is important to understand economic developments in Luxembourg. For instance, since cross-border workers account for a significant share of the labor income that is mostly spent abroad, domestic demand management may be more challenging than in other countries and fiscal policy may have different effects.

To take into account these potentially important mechanisms, we introduce cross-border workers in LU-EAGLE. To be more specific, we make foreign labor a required input in the production functions of firms located in the Luxembourg block of the model. As in the original EAGLE, these firms produce tradable and nontradable intermediate goods from labor (and capital) services. In LU-EAGLE, we assume that these labor services are given by

$$N_t = \left\{ (1-\omega_{CB})^{\frac{1}{\eta_{CB}}} \left[N_{R,t} \left(1-\Gamma_{R,t}\right)\right]^{\frac{\eta_{CB}-1}{\eta_{CB}}} + \omega_{CB}^{\frac{1}{\eta_{CB}}} \left(N_{CB,t}\right)^{\frac{\eta_{CB}-1}{\eta_{CB}}} \right\}^{\frac{\eta_{CB}}{\eta_{CB}-1}}, \tag{4}$$

where N_R stands for labor supplied by resident households, N_{CB} stands for labor supplied by cross-border households from the rest of the euro area, and $\omega_{CB} \in [0,1]$ and $\eta_{CB} > 0$ parametrize the relative weight of resident to foreign labor services and their elasticity of substitution. We also introduce an adjustment cost on resident labor, Γ_R, to help the model capture the higher volatility of cross-border labor observed in the data. This cost is given by

$$\Gamma_{R,t} = \frac{\gamma_R}{2} \left(\frac{N_{R,t}^D}{\overline{N}_R^D} - 1 \right)^2 \tag{5}$$

with $\gamma_R > 0$ and $\overline{N}_R^D$ the steady-state level of resident labor.

Regarding wage setting, we keep the original assumption that resident workers set their nominal wage rate subject to Calvo frictions and we need another condition to determine the wage of cross-border workers. The standard DSGE approach would be to introduce these agents explicitly in the model and to let them set their wage in the same fashion as resident workers. While more elegant, this approach would require cross-border workers to base their choices on variables from the block representing the rest of the euro area,[7] and it is unlikely that these would accurately capture the economic conditions relevant to workers from the Greater Region. Also, the wage received by cross-border workers would respond to fluctuations in the rest of the euro area and would have dynamics different from that received by resident workers, while the data suggests similar cyclical patterns in compensation per person for resident and foreign workers. Therefore, we close the labor market in the Luxembourg block of the model with the condition

$$W_{CB,t} = \kappa_{CB} W_{R,t}, \tag{6}$$

[7]A standard wage-setting equation would relate the wage of cross-border workers to aggregate consumption in the rest of the euro area. If cross-border workers could also choose whether they prefer to work in Luxembourg or in the block they live in, the aggregate wage rate in the rest of the euro area would also affect their optimal wage.

where $\kappa_{CB} > 0$. This formulation allows one to calibrate the relative wage of cross-border workers and to ensure that the wages of resident and cross-border workers share similar dynamics.

We emphasize two further adjustments linked to the introduction of cross-border workers. First, in line with the current institutional framework, we assume that cross-border workers pay income taxes directly in Luxembourg. They may also receive transfers from the Luxembourg state. Second, these wages and transfer payments are reflected in the current account of the Luxembourg and the rest-of-euro-area blocks of the model.[8] All details are provided in Appendix C.

3. Calibration

We calibrate the model at the quarterly frequency using a standard two-step strategy. First, we pin down values for the subset of parameters that govern the steady state of the model by matching sample averages extracted from national accounts and balance-of-payments data. Second, we choose values for the remaining parameters, which determine the dynamics of the model, by drawing heavily on earlier versions of EAGLE. Tables 1 – 5 report the outcomes of this approach.

Table 1 reports the values of several key ratios in the Luxembourg block of LU-EAGLE at the steady state of the model. The table compares these model-implied values with data averages computed over the 2010-2016 period using national accounts published by STATEC, the national statistical institute. Finally, the table reports the share of world GDP accounted for by Luxembourg in the model, as well as its empirical counterpart computed from data compiled by the World Bank. Overall, LU-EAGLE provides a satisfactory match to the structure of the Luxembourg economy. In particular, it reproduces both the expenditure and income decompositions of GDP, capturing the openness of the economy and the significant role of cross-border workers in the labor market. The model also captures the structure of public finance, as the shares of public expenditure and revenue in GDP are correctly reproduced.

The statistics reported in the left-hand column of Table 1 are endogenous model outcomes, so most of them are not directly calibrated.[9] Hence, we now review the actual parameter values used in LU-EAGLE.

Table 2 reports the parameters related to the behavior of households and firms. Regarding households, we assume identical preferences across types and regions and choose standard parameter values for the subjective discount factor, the intertemporal elasticity of substitution, the degree of consumption habits, the labor supply elasticity, and the depreciation rate of capital. As in Gomes, Jacquinot, and Pisani (2012), we assume that 25% of households are liquidity constrained in each block. Also in line with the original

[8] They also figure on the revenue side of households' budget constraints in the rest-of-euro-area block.

[9] Only the share in world GDP, as well as some fiscal targets, appear as explicit parameters that can be calibrated directly.

version of EAGLE, we set labor-market markups to 30% in the euro-area blocks of the model, above their value in the United States and the rest of the world.

In the Luxembourg block, the presence of cross-border workers requires the calibration of additional parameters. First, from the observed average share of cross-borders workers in total employment (42.5%) and in the aggregate wage bill (40%), we deduce that the average wage earned by cross-border worker is equal to 90% of that earned by resident workers.[10] This allows us to pin down κ_{CB} in equation (6). Then we fix the bias toward cross-border workers — ω_{CB} in equation (4) — to 0.41 so as to match the share of the aggregate wage bill going to cross-border workers. Finally, we assume that resident and non-resident workers provide labor services that are complement in production, implying a less-than-unity elasticity of substitution η_{CB}, to capture the idea that cross-border workers provide services for which no good local substitutes exist.

In the production function, we assume a larger capital share and a smaller markup in the tradables sector compared to the nontradables sector. This calibration emphasizes that the nontradables sector is typically more labor intensive and less competitive than the tradables sector. Regarding the production of final goods, we assume that tradable and nontradable inputs are complements and that domestic and imported tradables are substitutes, as in the benchmark EAGLE. The various CES weights are calibrated to be consistent with available estimates the share of nontradables and the level of import content in each of the four blocks.[11]

Table 3 presents the parameters related to international trade. We assume an elasticity of substitution between bloc-specific imports of 2.50, and we set the bilateral import weights to match a trade matrix estimated over 2010-2016 using various sources (STATEC for Luxembourg series, Eurostat for the rest-of-euro-area block, and the Census Bureau for the United States). Unsurprisingly, the rest of the euro area is Luxembourg's major trading partner: it accounts for 85% of its imports and 70% of its exports. It is also worth noting that in LU-EAGLE, economic interactions within the monetary union are almost one-sided. Given the small size of Luxembourg compared to the rest of the euro area, shocks originating from the Luxembourg block have only negligible effects on economic developments in its trading partners.

Table 4 reports the parameters characterizing the real and nominal rigidities that determine model properties in dynamic simulations. As in the original version of EAGLE, we assume a large cost of changing the degree of capital utilization, which essentially prevents firms from varying input utilization in the short run. Investment and imports

[10]STATEC (2017, section 6.2) explains the lower wage of cross-border workers by differences in qualification, job characteristics, and especially sectors of activity. In particular, resident workers are much more likely to work in the public sector.

[11]For Luxembourg, we use the input-output tables published by the STATEC between 2010 and 2016 to estimate the nontradable and import contents of final goods. Regarding import content, we use the following values: 49% for private consumption, 19% for public consumption, 64% for investment, and 68% for exports. For the other three blocks, we do not have precise data for these shares and instead use estimates from various sources, including the OECD (2011) and Lombardo and Ravenna (2012).

are subject to standard adjustment costs, and we set the adjustment costs on resident labor services in Luxembourg to a small value, which appears sufficient to capture the relative volatility of resident and cross-border labor. The negligible adjustment cost on foreign assets is required to ensure a well-defined steady state in the model.

Regarding nominal frictions, we also choose parameter values similar to those used in the baseline EAGLE. In the two euro area blocks, including Luxembourg, we set the Calvo price coefficients for domestic tradable and nontradable intermediate goods to 0.90, in line with the estimates reported by Smets and Wouters (2003) and Warne, Coenen, and Christoffel (2008) for the euro area.[12] In the other two blocks, the Calvo price coefficients are set to 0.75, implying an average price duration of one year. All Calvo wage parameters are also set to 0.75.[13] Lastly, to generate persistence we set the indexation parameter on prices to 0.50 and the one on wages to 0.75.[14]

Finally, in Table 5 we report the parameter values for the monetary and fiscal policy rules. With respect to monetary policy, we assume that identical Taylor-like rules are implemented in the euro area, the United States, and the rest of the world. All blocks share a common inflation target of 2% per year, and the policy rates react to their lagged value, annual inflation, and the quarterly output gap with elasticities that are standard in the literature. In the euro area, monetary policy reacts to union-wide variables.[15]

As for fiscal policy, the average tax rates for Luxembourg are chosen to match the observed revenue shares reported in Table 1. Overall, most rates seem well in line with empirical evidence. In particular, the consumption tax rate is set to 16%, close to Luxembourg's standard VAT rate of 17%, and the employer and employee social contribution rates are set to 15% and 13%, also relatively close to the legal rates. Comparison with the data is more difficult for the labor, capital, and profit tax rates, which must be inferred from taxes on income, wealth, and production. Our estimate of the labor tax

[12]Lunnemann and Matha (2005) estimate a monthly frequency of price change in Luxembourg of 12% (excluding temporary sales), which would correspond to a quarterly Calvo parameter of 0.60. Such a value would fail to generate sufficient price inertia in the model. Once energy and food prices are removed from their sample, the average frequency of price change drops around 7%, which would be consistent with a Calvo coefficient of 0.75. As is well known, DSGE models often require even higher Calvo coefficients to deliver plausible inflation responses.

[13]Using micro data, Lunnemann and Wintr (2009) estimate that each month about 7% of wages are adjusted in Luxembourg, excluding the automatic effects of institutional changes such as indexation. Converted to a quarterly figure, this corresponds to a Calvo wage coefficient of 0.75.

[14]Following standard terminology in the DSGE literature, we use the word 'indexation' to designate the direct effect of consumer inflation from the previous quarter on current price and wage developments. This should not be confused with Luxembourg's legal indexation mechanism that automatically adjusts wages, pensions, and social benefits to past inflation. Indeed, the legal mechanism operates according to a threshold system, which triggers only when the cumulative increase in consumer prices reaches 2.5%. This kind of non-linearity is difficult to capture in DSGE models, in which the feedback from past inflation occurs at each period.

[15]Of course, Luxembourg has a negligible contribution to aggregate euro-area data.

rate, set to 16%, is slightly below the implicit rate of about 20% reported by the European Commission (2016) for Luxembourg. As for the capital and profit taxes, the values we obtain are roughly in line with those in Deák, Fontagné, Maffezzoli, and Marcellino (2012) and in Marchiori and Pierrard (2015), who calibrated other DSGE models for Luxembourg. We estimate the share of public transfers to cross-border households from balance-of-payments data. For the three other blocks, we calibrate the tax rates using existing studies (Gomes, Jacquinot, and Pisani, 2012; Jacquinot, Pisani, Gomes, Bokan, and Gerali, 2016). As in the original version of EAGLE, lump-sum taxes adjust slowly to changes in the debt-to-GDP ratio to avoid explosive dynamics.

4. Quantitative Experiments

To illustrate the workings of LU-EAGLE, this section presents a set of simulations that describe the model's responses to several standard shocks. Since no other DSGE model for Luxembourg includes monetary policy, we first consider an expansionary monetary policy shock in the euro area. Then, we discuss a foreign shock that exogeneously boosts aggregate demand in the rest of euro area, in order to evaluate the model's ability to propagate international shocks. Finally, we describe the effects of an array of domestic policy shocks in Luxembourg that affect government expenditures, labor income taxes, and capital income taxes. Throughout, we highlight how the main characteristics of Luxembourg's economy shape the model's responses. Except for the monetary policy shock, whose persistence is endogenously determined by the policy rule, we assume a persistence parameter of 0.95 on all the other shocks considered.

4.1. **Monetary policy shock in the euro area.** Figure 1 shows the responses of key economic variables in Luxembourg following a monetary policy shock in the euro area that triggers a decline in the annualized nominal policy rate of about 100 basis points on impact. The policy rule governs the resulting dynamics of the nominal interest rate, which progressively returns to its baseline level and reaches it after about two years.

The responses of the other variables are largely standard. Because prices are sticky in the short run, the reduction in the nominal rate lowers the *ex ante* real interest rate, stimulating domestic consumption and investment in Luxembourg. Similar mechanisms apply in the rest of the euro area, in which aggregate demand also rises. This has a positive effect on Luxembourg's exports, which are largely biased toward other euro area countries and are driven by imports in the rest of the euro area in the simulation. Because of the high import content of final production in Luxembourg, imports rise as well. Nevertheless, the net effect on the trade balance is positive, representing about 0.6% of steady-state GDP on impact. Overall, real GDP in Luxembourg increases by up to a maximum of 0.9% immediately after the shock, before returning to its long-run level after about 4 years.

In order to accommodate the sustained increase in aggregate demand, firms use more labor (capital is inelastic in the short run). As a result, both hours worked and the real wage rate rise in Luxembourg, with labor being supplied by both resident and cross-border workers. Because of the adjustment cost, the response in cross-border labor is about three times larger than the response in resident labor. Higher wages also translate into higher production costs, so that firms progressively increase their prices. This generates a hump-shaped response of CPI inflation, which peaks after two quarters just below 0.4 percentage point.

As for international variables, lower interest rates in the euro area cause a depreciation in the effective exchange rates of the two euro blocks (one unit of domestic good exchanges against a smaller amount of foreign goods), while higher export prices translate into an appreciation of the terms of trade.[16] During the first year after the shock, sustained economic activity generates additional tax revenue for the government, representing about 0.3% of steady-state GDP at the peak. As public expenditures are unchanged, the primary deficit falls by a similar amount.

It is also interesting to contrast the effects of the monetary policy shock in Luxembourg and in the rest of the euro area. Overall, the responses are fairly similar in the two blocks, but there are two noticeable differences. First, while the maximal effect on real GDP occurs on impact in Luxembourg, it is delayed by 3 quarters in the rest of the euro area, where it is also smaller in relative terms (0.6 points of GDP, versus 0.9 points in Luxembourg). Second, in the short run CPI inflation rises by about 0.1 percentage point more in Luxembourg than in the rest of the euro area, implying that the real interest rate falls more in Luxembourg after the shock. These differences have a common explanation: international spillovers are much stronger in Luxembourg. Indeed, while the monetary shock triggers similar declines in the real interest rate in both blocks, Luxembourg also benefits from a higher boost to foreign demand than the rest of the euro area (as is most clearly seen from the response of the trade balance), which puts further upward pressure on GDP and inflation. Hence, the comparison emphasizes that the calibration of the size of the Luxembourg block and the detailed modeling of trade linkages in LU-EAGLE are quite important to properly evaluate the effects of the monetary policy shock on Luxembourg's economy.

4.2. **Foreign demand shock.** Figure 2 shows the effects of a preference shock that drives up consumption in the block representing the rest of the euro area. The shock is normalized to generate an immediate one percent increase of real GDP in the rest of the euro area.[17] Compared to the monetary policy shock, which drove down interest rates in both euro-area blocks, this foreign shock does not have a direct effect on Luxembourg's economy. Instead, it only propagates to the domestic economy through international

[16]In the model, bilateral terms of trade are defined as the ratio between the domestic price of imports and the price of exports converted to the domestic currency. Hence, a rise in the terms of trade signals that imports become relatively more expensive, which signals a deterioration.

[17]Since preferences are unit free, we follow Gomes, Jacquinot, and Pisani (2012) in calibrating the shock size to obtain the desired impact on GDP.

linkages, allowing to evaluate the strength of this spillover channel for Luxembourg in LU-EAGLE.

More specifically, the foreign shock affects economic dynamics in Luxembourg through trade in goods and assets. Since Luxembourg's exports are largely biased toward the other euro area countries, real exports increase as foreign demand rises, up to 1.2% one year after the shock. Since imports are required in production, they also rise. The net effect on the trade balance is positive and amounts to about 1% of steady-state GDP, which is enough to sustain an expansion of real GDP over about two years. Equilibrium employment also rises and, as in the previous experiment, both resident and cross-border workers contribute to this upward movement. As usual, higher labor demand translates into higher wages and finally higher prices, so that CPI inflation increases over five quarters. There is also a mechanical increase in tax revenues and the primary deficit falls by about 0.15% of GDP.

At the same time, the model finds protracted falls in both consumption and investment. This is explained partly by trade in assets: as households in the rest of the euro area become more willing to consume, they borrow on international markets, and households in Luxembourg cut domestic expenditures to lend, increasing the country's net asset position. The rise in the euro area policy rate, in response to euro area inflation and output developments, puts further downward pressure on domestic demand in the medium term, as it is associated with higher real rates. It also triggers an immediate appreciation of the euro against other currencies, which results in a fall of the real effective exchange rate. The latter then increases over time, as the euro depreciates because of the high demand addressed to the other blocks and of higher inflation.

This simulation illustrates how LU-EAGLE captures the propagation of shocks from one block to another. To model a small open economy like Luxembourg, which is highly dependent on foreign demand and therefore potentially exposed to international shocks, this constitutes a very attractive feature.

4.3. **Government expenditure shock.** Figure 3 shows the effects of an exogenous increase in public expenditures in Luxembourg, equal to one percent of *ex ante* GDP.

Real GDP increases by about 0.75% on impact, which corresponds to a multiplier slightly below 0.80, before slowly returning to its steady-state level as the effects of the shock vanish. At longer horizons, taking into account the dynamics of both GDP and government expenditures, the average multiplier is equal to 0.73 after one year, 0.65 after two years, and converges to 0.50 in the long run.[18] It follows that, according to LU-EAGLE, the government spending multiplier in Luxembourg is below one at all horizons. This pattern suggests that public expenditure does not provide a very effective channel

[18]The impact multiplier can be interpreted as follows: for each additional euro of public expenditures, GDP increases on impact by 0.80 euros. At longer horizons, the multipliers have a similar interpretation.

to stimulate private demand in Luxembourg.[19] Also, the fiscal multipliers in LU-EAGLE are largely in line with BCL estimates from structural vector auto-regressions.

In the model, the expenditure shock triggers a persistent fall in private consumption as well as in investment, after a short-lived increase to build up the capital stock.[20] This suggests some crowding out of private demand, largely through the negative wealth effect experienced by resident households. Indeed, rational and forward-looking agents anticipate that the government will increase taxes in the future to respect its intertemporal budget constraint. Therefore, they acknowledge that the net present value of their lifetime wealth drops after the shock and households react by cutting expenditures. On the other hand, the government spending shock is associated with a positive substitution effect: as real wages and inflation rise in Luxembourg, the near-zero response of the euro-area nominal policy rate implies a fall in the real interest rate, which acts to support aggregate demand in Luxembourg.

Because of the economic openness of Luxembourg, the government expenditure shock also has strong effects on trade variables. On the one hand, the rise in public demand and GDP is associated with higher imports, required to raise output. On the other hand, exports fall for two reasons. First, production moves from the tradables to the nontradables sector, as government consumption contains a large nontradables component. This reduces resources in the tradables sector, limiting production of exports. Second, domestic prices rise after the shock, as can be seen from the response of CPI inflation, raising export prices and progressively shifting foreign demand away from Luxembourg. The net effect on trade is negative, with the trade balance falling by 0.2-0.3% of steady-state GDP during several quarters. Of course, given the small size of Luxembourg, spillovers in the three other blocks are negligible.

In terms of public finances, there are also opposing forces at play. By assumption, government expenditures increase by 1% of steady-state GDP after the shock, before slowly returning to their long-run level. At the same time, the rise in economic activity, including hours worked by resident and cross-border workers, provides the government with higher tax revenues, which represent about 0.2% of steady-state GDP on impact. The net effect of the spending shock on the public budget is negative, with the primary deficit increasing immediately by 0.8% of steady-state GDP before gradually returning to baseline.

We close the section with a quick discussion of the effects of government spending shocks in the rest of the euro area (not reported in Figure 3). While the impact spending multiplier is larger in the rest of the euro area (0.86, versus 0.80 for Luxembourg), the

[19]This is consistent with the findings of the empirical literature on fiscal multipliers, which identifies trade openness as a critical determinant for the size of spending multipliers (Ilzetzki, Mendoza, and Végh, 2013). More open economies are associated with smaller multipliers, so it is not surprising that multipliers are smaller than one in Luxembourg.

[20]The initial rise in investment is largely driven by the fall in the real interest rate resulting from higher inflation.

medium-run effects on real GDP are virtually identical in the model: at the two-year horizon, the multipliers are equal to 0.65 in both blocks. However, these similar responses arise from different effects on the subcomponents of GDP. In particular, the trade balance falls more in Luxembourg (0.3 points of GDP, versus less than 0.1 points in the rest of the euro area), while private investment falls more in the rest of the euro area (-0.8%, versus -0.1% in Luxembourg). We interpret these results as suggesting that Luxembourg's degree of economic openness is important to understand the propagation of domestic shocks.

4.4. **Tax shocks.** Finally, Figures 4 and 5 show the effects of exogenous increases in the labor income tax rate and the capital income tax rate in the Luxembourg block. We assume that the capital tax shock also increases the taxation of corporate profits. Both shocks are normalized to generate a rise in tax revenues equal to one percent of *ex ante* GDP, which corresponds to a 2-point increase in the tax rates.[21]

Both shocks negatively affect real GDP in Luxembourg and their effects tend to build up over several years. Indeed, while GDP barely responds on impact, it falls by up to 0.2% after 16 quarters (4 years) following a shock to labor income tax, and by up to 0.15% after 24 quarters (6 years) following a shock to capital income tax. Therefore, the model suggests that the aggregate effects of tax shocks in Luxembourg, when summarized by the behavior of real GDP, are larger in the medium run than at short horizons. This is also reflected in the implied tax multipliers, whose values increase (in absolute terms) over several years: for the labor tax shock they are -0.04 after one year, -0.08 after two years, and -0.18 after four years, while for the capital tax shock they are -0.03 after one year, -0.07 after two years, and -0.15 after four years.[22] Accordingly, in LU-EAGLE tax multipliers are lower than spending multipliers, which is in line with standard Keynesian theory (after a tax cut households may choose to save a substantial fraction of the additional after-tax income, while additional government spending appears directly in aggregate demand), as well as results from other DSGE models (Coenen et al., 2012).[23]

Although the effects of the shocks on real GDP strengthen gradually, some subcomponents of aggregate demand react faster. Indeed, private consumption falls relatively quickly after the shock to labor income taxation and reaches its trough after roughly a

[21]Since government expenditures are fixed in this simulation, movements in the primary deficit track those in tax revenues.

[22]The first multiplier can be interpreted as follows: for each additional euro of tax revenues raised by the government over a one year period through higher labor income taxes, GDP falls on average by 0.04 euros. The other multipliers have similar interpretations.

[23]The model also suggests that raising capital taxes entails slightly less distortions than raising labor taxes, given that the associated multipliers are smaller. However, we do not view this as a very robust finding, for at least two reasons. First, as in most DSGE models, LU-EAGLE features adjustment costs on capital but not on labor, an asymmetry implying that labor will adjust more quickly than capital after a tax shock. In practice, adjustment lags of various types may slow the response of labor and lower labor tax multipliers. Second, physical capital does not reallocate between blocks in LU-EAGLE, as bonds are the only internationally traded assets. In practice, it is likely that after a tax shock some capital will be internationally redistributed, which would increase the capital tax multiplier.

year, while investment starts recovering two years after a capital tax shock. These dynamics are largely due to the negative wealth effect experienced by resident households, who recognize that higher taxes reduce the current value of their lifetime wealth. Both consumption and investment eventually return to their long-run levels. The two tax shocks also have adverse effects on hours worked, partly because households optimally choose to cut labor supply in response to higher taxes, and partly because firms demand less labor as they lower production.

As for international trade, the decline in production is associated with a drop in imports, which improves the trade balance in the short run. At longer horizons, both tax shocks trigger a persistent slump in exports, which ends up being the main contributor to the medium-term fall in real GDP. This downward movement in exports is caused by the loss of competitiveness experienced by domestic firms, resulting from a rise in wages (after a labor tax shock) or in capital costs (after a capital tax shock) that increases marginal costs and prices. This can be seen most clearly from the response of CPI inflation, which is positive in the medium run after both shocks. These simulations provide further support for the claim that the trade channel is key to understanding the impact of fiscal shocks in Luxembourg.

Interestingly, tax multipliers in the model are much larger (in absolute value) in the rest of the euro area than in Luxembourg. For instance, at the one year horizon, the labor tax multiplier in the block representing the rest of the euro area is equal to -0.36 and the capital tax multiplier is equal to -0.15. This marked difference originates from two specific features of Luxembourg's economy. First, with about 40% of labor income being paid to cross-border workers and spent outside the country, a rise in labor taxes has less negative effects on domestic demand in Luxembourg than in other countries. Second, given the openness of the economy, aggregate dynamics in Luxembourg are less dependent on domestic factors than in countries that are more closed. Once more, this discussion highlights the need to adapt the model to Luxembourg when assessing the effects of various shocks.

5. Conclusion

In this paper, we have described LU-EAGLE, a DSGE model developed at the Banque centrale du Luxembourg to support short- and medium-run economic analysis. LU-EAGLE builds on the ESCB's EAGLE model, from which it borrows its general structure, including its formalization of the euro area as a currency union. It also embeds two specific features required to match some key characteristics of Luxembourg's economy: the high levels of exports and imports relative to GDP and the significant contribution of cross-border workers to the labor force. We have calibrated the model and analyzed the effects of a set of standard shocks originating both domestically and abroad.

We conclude by highlighting some possible directions for further research. Given the large share of financial services in Luxembourg's GDP, it may be interesting to introduce these explicitly in the model, for instance along the lines of Marchiori and Pierrard (2015).

This would help clarify the role of the financial sector in driving economic fluctuations in Luxembourg, either as a source of shocks or as an amplification channel. Another interesting avenue may be to improve the quantitative side of the model through structural estimation. However, this would probably require simplifying the model by making the three non-Luxembourg blocks exogenous.

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Appendix A. Tables and Figures

Table 1. Key ratios in Luxembourg — Steady-state values in the model vs. data.

	Model	Data
GDP decomposition (expenditure approach)		
Private consumption	0.32	0.32
Private investment	0.20	0.19
Public consumption	0.17	0.17
Exports	1.96	1.96
Imports	1.65	1.65
GDP decomposition (income approach)		
Labor income	0.49	0.49
To resident workers	0.29	0.29
To cross-border workers	0.20	0.20
Capital income (including profits)	0.51	0.51
Public finance		
Expenditures	0.39	0.39
Public consumption	0.17	0.17
Transfers	0.22	0.22
Revenues	0.40	0.40
VAT-like tax	0.11	0.11
Social security contributions	0.12	0.12
Labor income tax	0.07	0.07
Capital income tax	0.10	0.10
Primary deficit	-0.01	-0.01
Size		
Luxembourg share in world GDP	0.0005	0.0005

Notes. Except for 'size', all numbers represent shares in Luxembourg's nominal GDP. 'Data' refers to 2010-2016 sample averages extracted from the national accounts published by STATEC, except for labor and capital income taxes which come from the European Commission's AMECO database and the share in world GDP which comes from the World Bank. Because of rounding errors, the shares reported in the GDP decompositions may not sum exactly to 100%.

TABLE 2. Parameters determining household and firm behavior.

	LU	REA	US	RoW
Households				
Discount factor	0.995	0.995	0.995	0.995
Intertemporal elasticity of substitution	1.00	1.00	1.00	1.00
Inverse Frisch elasticity of labor supply	2.00	2.00	2.00	2.00
Habit persistence	0.60	0.60	0.60	0.60
Share of liquidity-constrained households	0.25	0.25	0.25	0.25
Depreciation rate	0.025	0.025	0.025	0.025
Wage markup	1.30	1.30	1.16	1.16
Labor market in LU				
Bias toward cross-border workers	0.41	—	—	—
Substitution b/w resident and cross-border workers	0.50	—	—	—
Relative wage of cross-border workers	0.90	—	—	—
Intermediate sector				
Capital bias in nontradables	0.30	0.30	0.30	0.30
Price markup for nontradables	1.50	1.50	1.30	1.30
Capital bias in tradables	0.35	0.35	0.35	0.35
Price markup for tradables and exports	1.30	1.30	1.20	1.20
Final sector				
Bias toward tradables in consumption	0.79	0.74	0.33	0.30
Bias toward tradables in government expenditures	0.45	0.35	0.11	0.10
Bias toward tradables in investment	0.84	0.74	0.48	0.46
Substitution b/w tradables and nontradables	0.50	0.50	0.50	0.50
Bias toward domestic tradables in consumption	0.37	0.76	0.51	0.45
Bias toward domestic tradables in government expenditures	0.55	0.72	0.51	0.45
Bias toward domestic tradables in exports	0.35	0.79	0.86	0.82
Bias toward domestic tradables in investment	0.21	0.79	0.91	0.64
Substitution b/w domestic and imported tradables	1.50	1.50	1.50	1.50

Notes. 'LU' stands for Luxembourg, 'REA' for the rest of the euro area, 'US' for the United States, and 'RoW' for the rest of the world.

Table 3. Parameters determining international linkages (import bundles).

	LU	REA	US	RoW
Substitution b/w imports	2.50	2.50	2.50	2.50
Bias toward imports from				
LU	—	0.086	0.006	0.016
REA	0.905	—	0.216	0.715
US	0.043	0.106	—	0.269
RoW	0.052	0.808	0.778	—

Notes. 'LU' stands for Luxembourg, 'REA' for the rest of the euro area, 'US' for the United States, and 'RoW' for the rest of the world.

Table 4. Parameters determining real and nominal rigidities.

	LU	REA	US	RoW
Adjustment costs				
Capital utilization	2000	2000	2000	2000
Investment	5.00	5.00	5.00	5.00
Imports	2.00	2.00	2.00	2.00
Resident labor force	1.00	—	—	—
Foreign asset position	0.001	0.001	0.001	0.001
Calvo parameters				
Prices of domestic intermediates	0.90	0.90	0.75	0.75
Prices of exports	0.75	0.75	0.75	0.75
Wages	0.75	0.75	0.75	0.75
Indexation parameters				
Prices	0.50	0.50	0.50	0.50
Wages	0.75	0.75	0.75	0.75

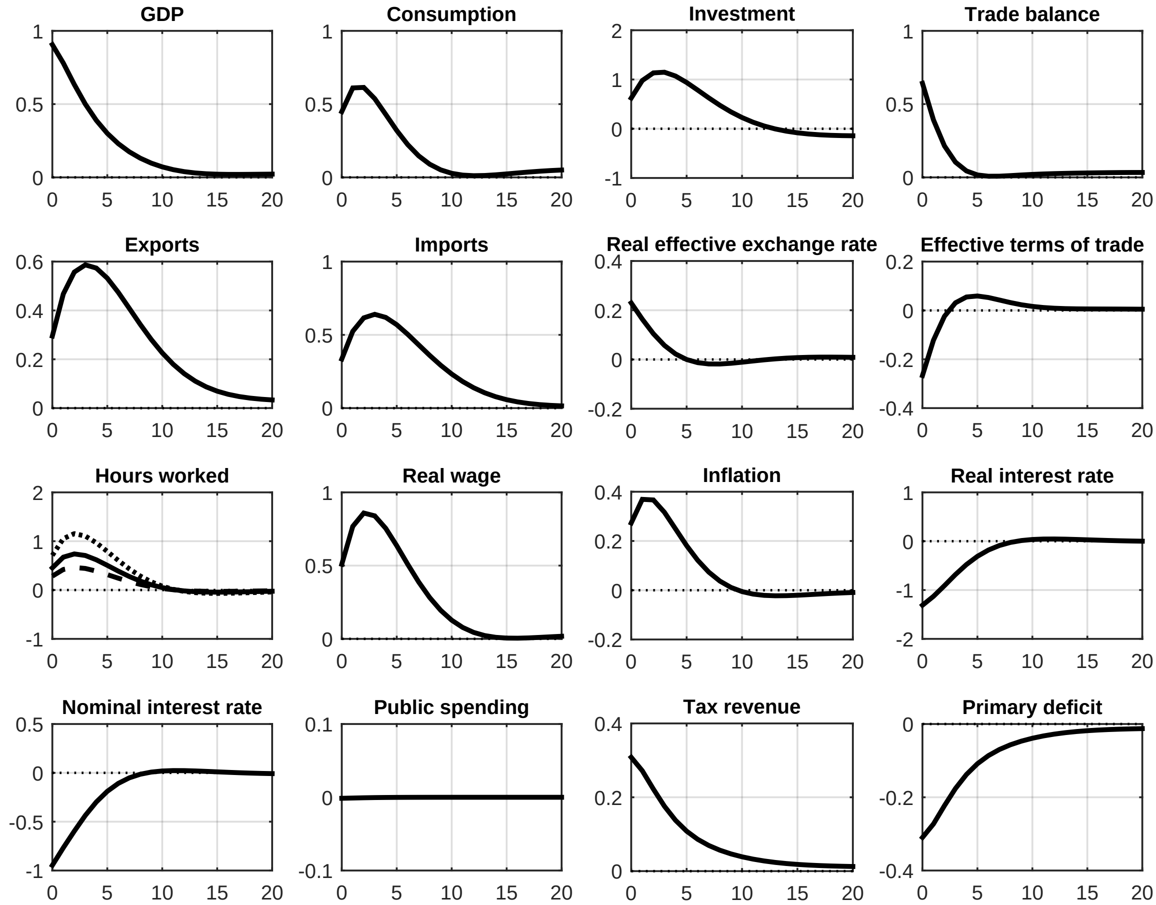
Notes. 'LU' stands for Luxembourg, 'REA' for the rest of the euro area, 'US' for the United States, and 'RoW' for the rest of the world.

Table 5. Parameters determining monetary and fiscal policy.

	LU	REA	US	RoW
Monetary policy				
Annual gross inflation target	1.02	1.02	1.02	1.02
Interest rate smoothing	0.85	0.85	0.85	0.85
Interest rate response to inflation	1.70	1.70	1.70	1.70
Interest rate response to output gap	0.20	0.20	0.20	0.20
Fiscal policy				
Share of transfers going to foreign households	0.19	—	—	—
Consumption tax rate	0.16	0.16	0.08	0.08
Labor income tax rate	0.16	0.15	0.15	0.15
Capital income tax rate	0.10	0.10	0.10	0.10
Profit tax rate	0.32	0.30	0.30	0.30
Rate of social security contributions by firms	0.15	0.22	0.08	0.11
Rate of social security contributions by households	0.13	0.12	0.07	0.08
Lump-sum taxes response to debt-to-output ratio	0.10	0.10	0.10	0.10

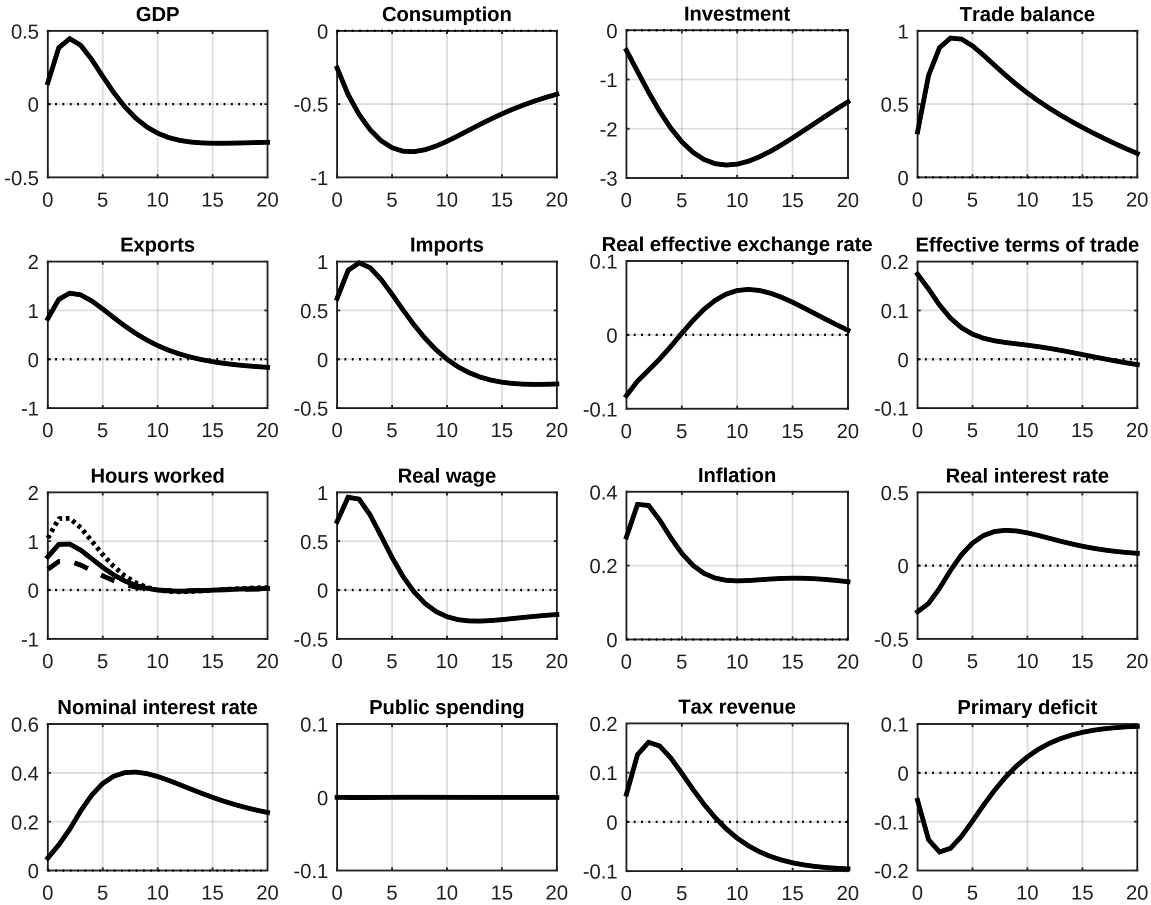
Notes. 'LU' stands for Luxembourg, 'REA' for the rest of the euro area, 'US' for the United States, and 'RoW' for the rest of the world. Remark that Luxembourg and the rest of the euro area share a common monetary authority.

FIGURE 1. Reduction in the euro area policy rate — Effects in Luxembourg.



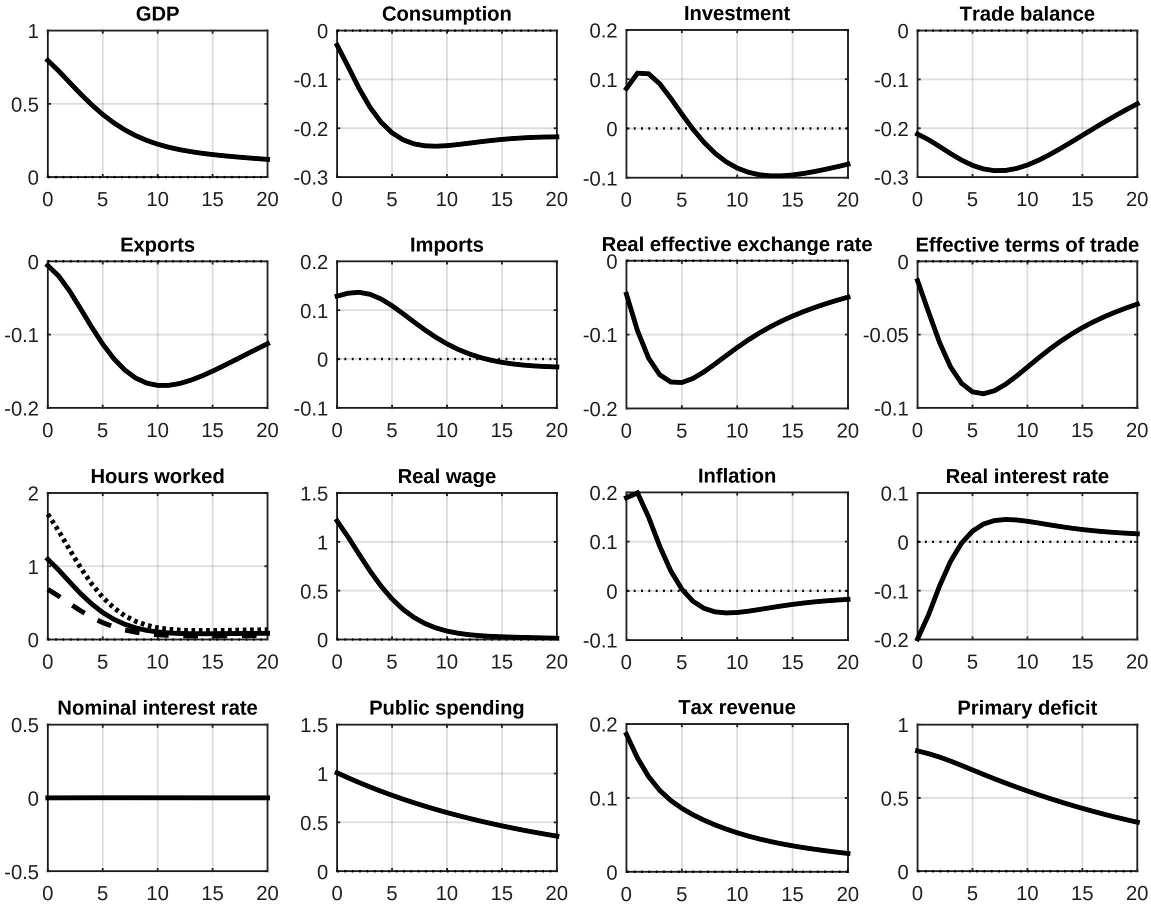
Notes. Horizontal axis: quarters. Vertical axis: % deviations from the steady state, except for the trade balance, public spending, tax revenue, and the primary deficit (%-points of steady-state GDP), as well as inflation and interest rates (annualized %-point deviations). A rise in the exchange rate (terms of trade) signals a depreciation (deterioration). For hours worked, the dashed line represents resident labor, the dotted line cross-border labor, and the solid line average labor.

FIGURE 2. Increase in euro area aggregate demand — Effects in Luxembourg.



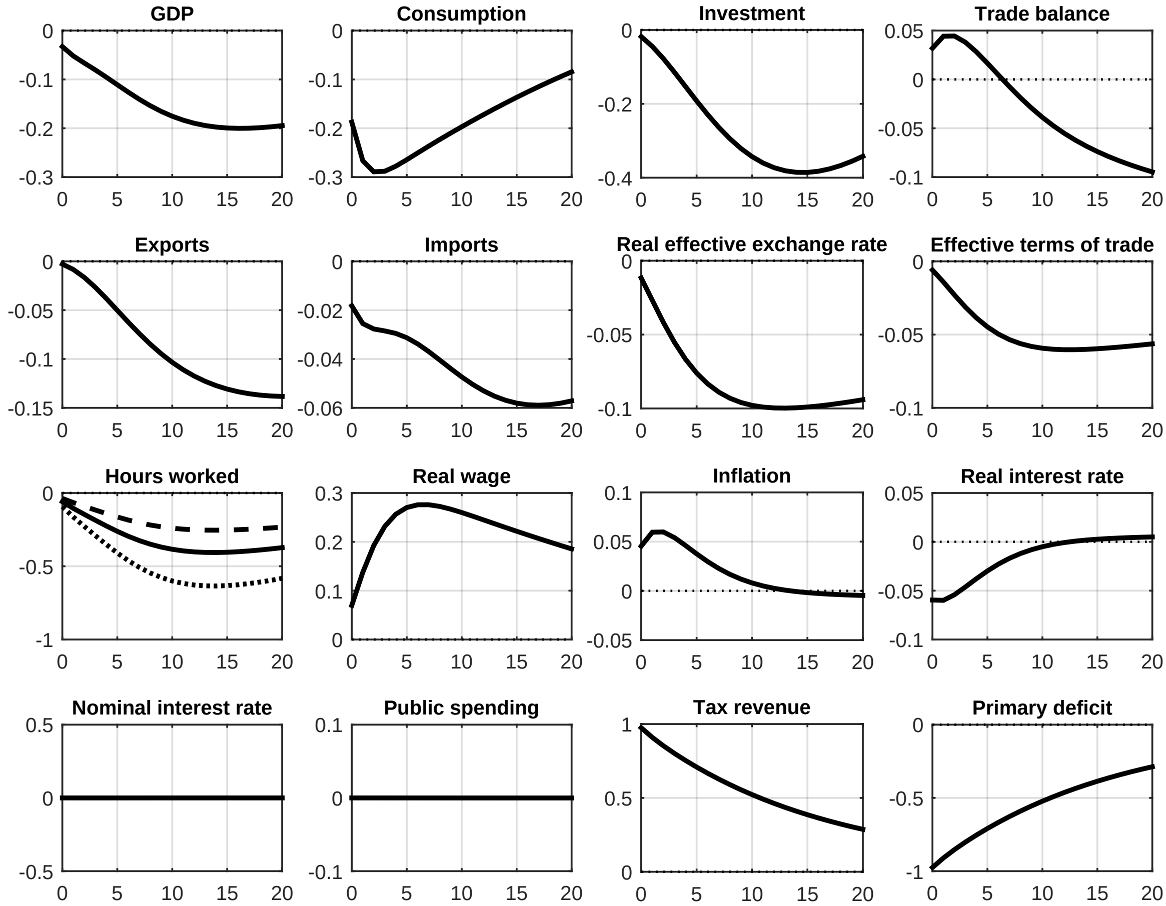
Notes. Horizontal axis: quarters. Vertical axis: % deviations from the steady state, except for the trade balance, public spending, tax revenue, and the primary deficit (%-points of steady-state GDP), as well as inflation and interest rates (annualized %-point deviations). A rise in the exchange rate (terms of trade) signals a depreciation (deterioration). For hours worked, the dashed line represents resident labor, the dotted line cross-border labor, and the solid line average labor.

FIGURE 3. Increase in government expenditures in Luxembourg — Effects in Luxembourg.



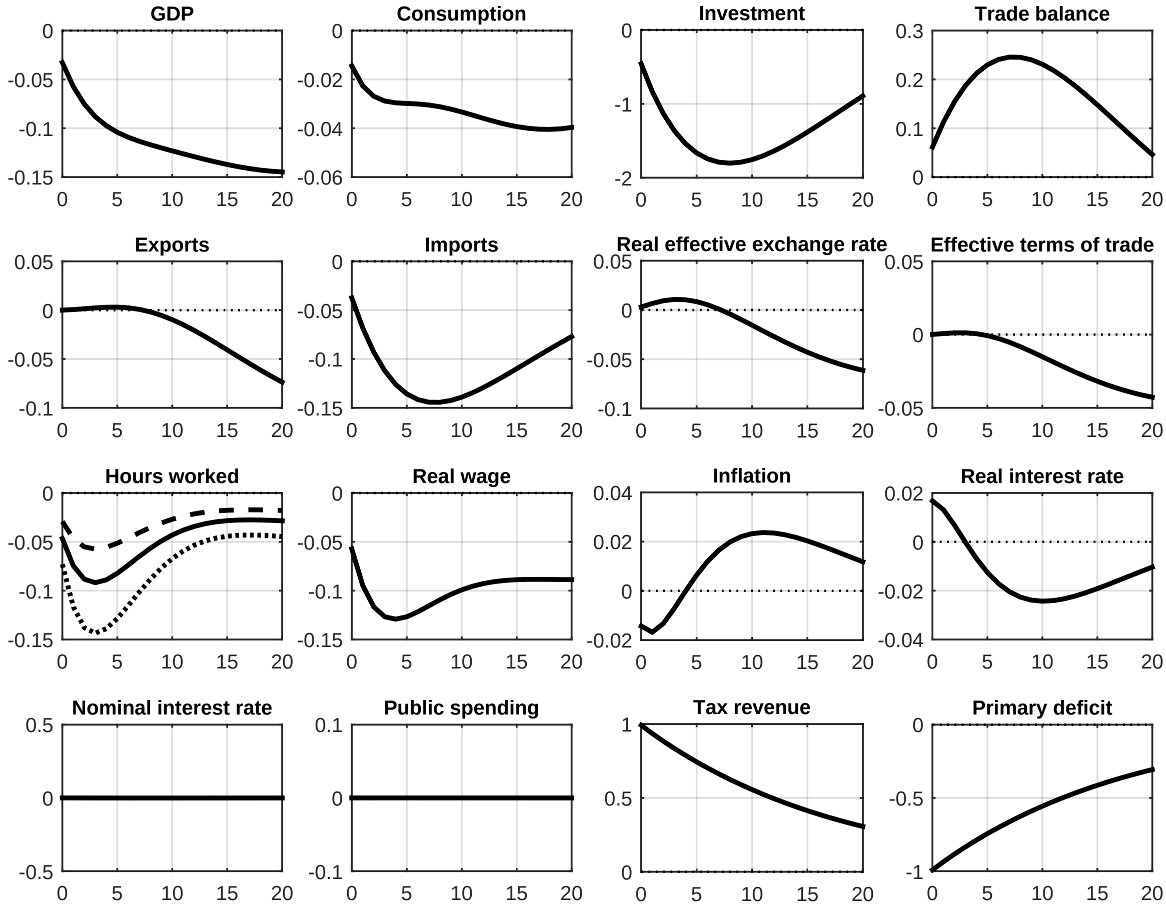
Notes. Horizontal axis: quarters. Vertical axis: % deviations from the steady state, except for the trade balance, public spending, tax revenue, and the primary deficit (%-points of steady-state GDP), as well as inflation and interest rates (annualized %-point deviations). A rise in the exchange rate (terms of trade) signals a depreciation (deterioration). For hours worked, the dashed line represents resident labor, the dotted line cross-border labor, and the solid line average labor.

FIGURE 4. Increase in the labor income tax rate in Luxembourg — Effects in Luxembourg.



Notes. Horizontal axis: quarters. Vertical axis: % deviations from the steady state, except for the trade balance, public spending, tax revenue, and the primary deficit (%-points of steady-state GDP), as well as inflation and interest rates (annualized %-point deviations). A rise in the exchange rate (terms of trade) signals a depreciation (deterioration). For hours worked, the dashed line represents resident labor, the dotted line cross-border labor, and the solid line average labor.

FIGURE 5. Increase in the capital income tax rate in Luxembourg — Effects in Luxembourg.



Notes. Horizontal axis: quarters. Vertical axis: % deviations from the steady state, except for the trade balance, public spending, tax revenue, and the primary deficit (%-points of steady-state GDP), as well as inflation and interest rates (annualized %-point deviations). A rise in the exchange rate (terms of trade) signals a depreciation (deterioration). For hours worked, the dashed line represents resident labor, the dotted line cross-border labor, and the solid line average labor.

Appendix B. Derivations: Import Contents

This appendix documents the derivations required to include the import content of government expenditures and exports in LU-EAGLE. It should be read together with the original model description provided in Gomes, Jacquinot, and Pisani (2012).

B.1. Government good.

B.1.1. *Production.* The government (G) good is produced in quantity $Q_t^G(x)$ by a representative firm $x \in [0, s^H]$ according to

$$Q_t^G(x) = \left[\nu_G^{\frac{1}{\mu_G}} TT_t^G(x)^{\frac{\mu_G - 1}{\mu_G}} + (1 - \nu_G)^{\frac{1}{\mu_G}} NT_t^G(x)^{\frac{\mu_G - 1}{\mu_G}} \right]^{\frac{\mu_G}{\mu_G - 1}},$$

where TT_t^G is the input basket of tradable goods specific to the G sector and NT_t^G is the input basket of domestic nontradable intermediates used in the G sector, with respective prices $P_{TT^G,t}$ and $P_{N,t}$. $\mu_G > 0$ measures the elasticity of substitution between tradable and nontradable inputs in the G sector, $\nu_G \in [0, 1]$ measures the relative weight of tradables in the G sector, and $s^H \in (0, 1)$ is the relative size of region H in the world economy. Cost minimization yields

$$TT_t^G(x) = \nu_G \left(\frac{P_{TT^G,t}}{P_{G,t}} \right)^{-\mu_G} Q_t^G(x),$$

$$NT_t^G(x) = (1 - \nu_G) \left(\frac{P_{N,t}}{P_{G,t}} \right)^{-\mu_G} Q_t^G(x),$$

and the associated price index for the government good

$$P_{G,t} = \left[\nu_G P_{TT^G,t}^{1-\mu_G} + (1 - \nu_G) P_{N,t}^{1-\mu_G} \right]^{\frac{1}{1-\mu_G}}.$$

B.1.2. *Production of the tradable input bundle.* The tradable input bundle specific to the G sector TT_t^G is produced according to

$$TT_t^G(x) = \left[\nu_{TG}^{\frac{1}{\mu_{TG}}} HT_t^G(x)^{\frac{\mu_{TG} - 1}{\mu_{TG}}} + (1 - \nu_{TG})^{\frac{1}{\mu_{TG}}} IM_t^G(x)^{\frac{\mu_{TG} - 1}{\mu_{TG}}} \right]^{\frac{\mu_{TG}}{\mu_{TG} - 1}},$$

where HT_t^G is the input basket of domestic tradables used in the G sector and IM_t^G is the input basket of imports specific to the G sector, with respective prices $P_{H,t}$ and $P_{IM^G,t}$. $\mu_{TG} > 0$ measures the elasticity of substitution between domestic and imported tradable inputs in the G sector, while $\nu_{TG} \in [0, 1]$ measures the weight of domestic tradables relative to imports in the G sector. Cost minimization implies

$$HT_t^G(x) = \nu_{TG} \left(\frac{P_{H,t}}{P_{TT^G,t}} \right)^{-\mu_{TG}} TT_t^G(x),$$

$$IM_t^G(x) = (1-\nu_{TG})\left(\frac{P_{IM^G,t}}{P_{TT^G,t}}\right)^{-\mu_{TG}} TT_t^G(x),$$

$$P_{TT^G,t} = \left[\nu_{TG}P_{H,t}^{1-\mu_{TG}} + (1-\nu_{TG})P_{IM^G,t}^{1-\mu_{TG}}\right]^{\frac{1}{1-\mu_{TG}}}.$$

B.1.3. *Production of the other input bundles.* The remaining input bundles, corresponding respectively to domestic nontradables, domestic tradables, and imports, are produced exactly as in EAGLE.

B.1.4. *Demand.* Nominal government expenditures evolve exogeneously according to

$$P_{G,t}G_t = (1-\rho_g)\overline{P}_G\overline{G} + \rho_g P_{G,t-1}G_{t-1} + \epsilon_{g,t},$$

where $\overline{P}_G\overline{G} > 0$ is the steady-state level of expenditures, $\rho_g \in [0,1]$ measures the persistence of public purchases, and $\epsilon_{g,t}$ is a serially uncorrelated public spending shock.

B.2. **Export good.**

B.2.1. *Production.* The homogeneous export (X) good is produced in quantity $Q_t^X(x)$ by a representative firm $x \in [0, s^H]$ according to

$$Q_t^X(x) = \left[\nu_{TX}^{\frac{1}{\mu_{TX}}} HT_t^X(x)^{\frac{\mu_{TX}-1}{\mu_{TX}}} + (1-\nu_{TX})^{\frac{1}{\mu_{TX}}} IM_t^X(x)^{\frac{\mu_{TX}-1}{\mu_{TX}}}\right]^{\frac{\mu_{TX}}{\mu_{TX}-1}},$$

where HT_t^X is the input basket of domestic tradables used in the X sector and IM_t^X is the input basket of imports specific to the X sector, with respective prices $P_{H,t}$ and $P_{IM^X,t}$. $\mu_{TX} > 0$ measures the elasticity of substitution between domestic and imported tradable inputs in the X sector, while $\nu_{TX} \in [0,1]$ measures the weight of domestic tradables relative to imports in the X sector. Cost minimization implies

$$HT_t^X(x) = \nu_{TX}\left(\frac{P_{H,t}}{P_{Q^X,t}}\right)^{-\mu_{TX}} Q_t^X(x),$$

$$IM_t^X(x) = (1-\nu_{TX})\left(\frac{P_{IM^X,t}}{P_{Q^X,t}}\right)^{-\mu_{TX}} Q_t^X(x),$$

$$P_{Q^X,t} = \left[\nu_{TX}P_{H,t}^{1-\mu_{TX}} + (1-\nu_{TX})P_{IM^X,t}^{1-\mu_{TX}}\right]^{\frac{1}{1-\mu_{TX}}}.$$

B.2.2. *Pricing.* Because it is produced under perfect competition, the homogeneous export goods cannot be associated with pricing frictions. To maintain sticky export prices in the model, we introduce a continuum of exporters that transform the X good into differentiated and specialized exports in a simple "brand naming" process à la Christiano, Trabandt, and Walentin (2011). Specialized export varieties, denoted generically ex, are thus produced according to

$$X_t(ex) = Q_t^X(ex) - \psi_X,$$

where $X_t(ex)$ stands for the production of export firm ex, $Q_t^X(ex)$ for its demand of the final export good, and $\psi_X \geq 0$ for a fixed production cost. It follows immediately that the marginal cost in the export sector is identical across firms and given by $MC_{X,t} = P_{Q^X,t}$, while nominal profits are

$$D_{X,t} = \sum_{CO \neq H} S_t^{H,CO} P_{X,t}^{H,CO} X_t^{H,CO} - P_{Q^X,t} Q_t^X.$$

Since each region faces three foreign blocks, export firms set three prices, each in the currency of the destination region (the local currency assumption holds). Individual export prices adjust sluggishly: for any given firm in the export sector, the probability of optimally resetting its prices at any given period is $1 - \xi_X$, with $\xi_X \in [0, 1]$. Prices that are not reoptimized adjust according to the indexation scheme:

$$P_{X,t}^{CO}(ex) = \left(\Pi_{X,t-1}^{H,CO}\right)^{\chi_X} \overline{\Pi}^{1-\chi_X} P_{X,t-1}^{CO}(ex),$$

where $\chi_X \in [0, 1]$ is the degree of price indexation for exports and $\Pi_{X,t-1}^{H,CO} = P_{X,t-1}^{H,CO} / P_{X,t-2}^{H,CO}$ is lagged average inflation for exports flowing from region H to region CO.

Each exporting firm that reoptimizes its prices at date t maximizes the expected discounted sum of its future nominal profits expressed in domestic currency and net of taxes:

$$E_t \sum_{k=0}^{\infty} \sum_{CO \neq H} \xi_X^k \left(1 - \tau_{t+k}^D\right) \Lambda_{I,t,t+k} \left\{ S_{t+k}^{H,CO} P_{X,t+k}^{CO}(ex) X_{t+k}^{CO}(ex) \right.$$
$$\left. - MC_{X,t+k} \left[X_{t+k}^{CO}(ex) + \psi_X \right] \right\},$$

where $S_t^{H,CO}$ is the nominal exchange rate between regions H and CO (that is, one unit of region CO's currency exchanges against $S_t^{H,CO}$ units of region H's currency). Maximization is subject to the above indexation rule and to the foreign demand for export variety ex:

$$X_{t+k}^{CO}(ex) = \left(\frac{P_{X,t+k}^{CO}(ex)}{P_{X,t+k}^{H,CO}} \right)^{-\theta_X} X_{t+k}^{H,CO}.$$

The optimal reset price for foreign market CO, $\widetilde{P}_{X,t}^{H,CO}$, is characterized by

$$E_t \sum_{k=0}^{\infty} \xi_X^k \left(1 - \tau_{t+k}^D\right) \Lambda_{I,t,t+k} \left[S_{t+k}^{H,CO} \prod_{s=1}^{k} \left(\Pi_{X,t+s-1}^{H,CO}\right)^{\chi_X} \overline{\Pi}^{1-\chi_X} \widetilde{P}_{X,t}^{H,CO} \right.$$
$$\left. - \frac{\theta_X}{\theta_X - 1} MC_{X,t+k} \right] X_{t+k}^{CO}(ex) = 0,$$

and it is the same for all firms in the export sector that reoptimize at date t. The bilateral price index of imports from region H to region CO $P_{X,t}^{H,CO}$ then evolves according to

$$P_{X,t}^{H,CO} = \left\{ \xi_X \left[\left(\Pi_{X,t-1}^{H,CO}\right)^{\chi_X} \overline{\Pi}^{1-\chi_X} P_{X,t-1}^{H,CO} \right]^{1-\theta_X} + (1 - \xi_X) \left(\widetilde{P}_{X,t}^{H,CO} \right)^{1-\theta_X} \right\}^{\frac{1}{1-\theta_X}}.$$

B.3. **Closing conditions.** Some closing conditions of the model need to be amended in view of the changes described above. The interested reader is referred to Gomes, Jacquinot, and Pisani (2012) for the definition of all newly introduced variables.

B.3.1. *Dividends.* One needs to take into account dividends from the export sector when defining aggregate dividends:

$$D_t = D_{H,t} + D_{N,t} + D_{X,t}.$$

B.3.2. *Market clearing.* The market-clearing condition for each domestic tradable good $h \in [0, s^H]$ is

$$Y_{H,t}^S(h) = HT_t(h).$$

Integrating the left-hand side over h gives per-capita aggregate supply in the tradable sector:

$$Y_{H,t}^S = \frac{1}{s^H} \int_0^{s^H} Y_{H,t}^S(h) dh.$$

Similarly integrating the right-hand side gives aggregate demand in the tradable sector:

$$\frac{1}{s^H} \int_0^{s^H} HT_t(h) dh = s_{H,t} HT_t,$$

where

$$s_{H,t} = \frac{1}{s^H} \int_0^{s^H} \left(\frac{P_t(h)}{P_{H,t}} \right)^{-\theta_H} dh$$

is a measure of price dispersion in the tradable sector. Market clearing for tradables then requires

$$\begin{aligned} Y_{H,t}^S &= s_{H,t} HT_t, \\ HT_t &= HT_t^C + HT_t^I + HT_t^G + HT_t^X. \end{aligned}$$

Similarly, market clearing for nontradables requires

$$\begin{aligned} Y_{N,t}^S &= s_{N,t} NT_t, \\ NT_t &= NT_t^C + NT_t^I + NT_t^G, \end{aligned}$$

while market clearing for exports requires

$$Q_t^X = \sum_{CO \neq H} s_{X,t}^{H,CO} \frac{s^{CO}}{s^H} \left(IM_t^{CO,H} + \psi_X \right),$$

where $IM_t^{CO,H}$ denotes total imports of region CO from region H.

Appendix C. Derivations: Cross-Border Workers

This appendix documents the derivations required to include the cross-border workers in LU-EAGLE.

C.1. **Production of intermediates.** In each region, intermediate firms produce differentiated tradable and nontradable goods under monopolistic competition, using labor and capital services as inputs. Each tradable variety is produced by a firm $h \in [0, s^H]$, while each nontradable variety is produced by a firm $n \in [0, s^H]$.

The production functions for tradable and nontradable intermediate varieties, denoted generically h and n, are given by

$$Y^S_{H,t}(h) = \max\left\{z_{H,t} K^D_{H,t}(h)^{\alpha_H} N^D_{H,t}(h)^{1-\alpha_H} - \psi_H, 0\right\},$$
$$Y^S_{N,t}(n) = \max\left\{z_{N,t} K^D_{N,t}(n)^{\alpha_N} N^D_{N,t}(n)^{1-\alpha_N} - \psi_N, 0\right\},$$

where $Y^S_{H,t}(h)$ and $Y^S_{N,t}(n)$ stand for the production levels of firms h and n, $K^D_{H,t}(h)$ and $K^D_{N,t}(n)$ for the capital services firms rent, and $N^D_{H,t}(h)$ and $N^D_{N,t}(n)$ for the labor services they rent. The parameters $\alpha_H, \alpha_N \in [0,1]$ measure technological bias toward capital in each sector, while $\psi_H, \psi_N \geq 0$ are fixed production costs. $z_{H,t}$ and $z_{N,t}$ are sector-specific technology processes.

C.2. **Definition of the labor input in the Luxembourg block.** In LU-EAGLE, labor services are defined as in EAGLE for the blocks corresponding to the rest of the euro area, the United States, and the rest of the world. In the Luxembourg block, however, labor services are defined as a bundle of differentiated services supplied by both resident and foreign households (from the rest-of-euro-area block).

Formally, the aggregate labor input used by the representative firm h from the tradable sector is defined as

$$N^D_t(h) = \left\{(1-\omega_{CB})^{\frac{1}{\eta_{CB}}} \left[N^D_{R,t}(h)\left(1-\Gamma_{R,t}(h)\right)\right]^{\frac{\eta_{CB}-1}{\eta_{CB}}} + \omega_{CB}^{\frac{1}{\eta_{CB}}} N^D_{CB,t}(h)^{\frac{\eta_{CB}-1}{\eta_{CB}}}\right\}^{\frac{\eta_{CB}}{\eta_{CB}-1}},$$

where $N^D_{R,t}$ stands for the demand of labor supplied by resident households, $N^D_{CB,t}$ stands for the demand of labor supplied by cross-border households, $\eta_{CB} > 0$ measures the elasticity of substitution between the two labor bundles, and $\Gamma_{R,t}$ is an adjustment cost on the demand for resident labor given by

$$\Gamma_{R,t}(h) = \frac{\gamma_R}{2}\left(\frac{N^D_{R,t}(h)}{\overline{N}^D_R} - 1\right)^2$$

with $\gamma_R > 0$. Letting $W_{R,t}$ and $W_{CB,t}$ denote the wage rates, cost minimization requires (in aggregate terms)

$$N^D_{R,t} = (1-\omega_{CB})\left(\frac{W_{R,t}}{W_t \Gamma^{\dagger}_{R,t}}\right)^{-\eta_{CB}} \frac{N^D_t}{1-\Gamma_{R,t}},$$

$$N^D_{CB,t} = \omega_{CB}\left(\frac{W_{CB,t}}{W_t}\right)^{-\eta_{CB}} N^D_t,$$

$$\Gamma^{\dagger}_{R,t} = 1 - \Gamma_{R,t} - \gamma_R\left(\frac{N^D_{R,t}}{\overline{N}^D_R} - 1\right)\frac{N^D_{R,t}}{\overline{N}^D_R}.$$

The bundle of resident labor $N^D_{R,t}(h)$ is defined as

$$N^D_{R,t}(h) = \left[(1-\omega)^{\frac{1}{\eta}} N^D_{I,t}(h)^{\frac{\eta-1}{\eta}} + \omega^{\frac{1}{\eta}} N^D_{J,t}(h)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

with

$$N^D_{I,t}(h) = \left[\left(\frac{1}{s^H(1-\omega)}\right)^{\frac{1}{\eta_I}} \int_0^{s^H(1-\omega)} N^D_t(h,i)^{\frac{\eta_I-1}{\eta_I}} di\right]^{\frac{\eta_I}{\eta_I-1}},$$

$$N^D_{J,t}(h) = \left[\left(\frac{1}{s^H\omega}\right)^{\frac{1}{\eta_J}} \int_{s^H(1-\omega)}^{s^H} N^D_t(h,j)^{\frac{\eta_J-1}{\eta_J}} dj\right]^{\frac{\eta_J}{\eta_J-1}}.$$

Cost minimization implies

$$N^D_{I,t} = \left(\frac{W_{I,t}}{W_{R,t}}\right)^{-\eta} N^D_{R,t},$$

$$N^D_{J,t} = \left(\frac{W_{J,t}}{W_{R,t}}\right)^{-\eta} N^D_{R,t},$$

$$N^D_{R,t} = \left[(1-\omega)\left(N^D_{I,t}\right)^{\frac{\eta-1}{\eta}} + \omega\left(N^D_{J,t}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

while the demand functions addressed to each household are

$$N^D_t(i) = \left(\frac{W_t(i)}{W_{I,t}}\right)^{-\eta_I} N^D_{I,t},$$

$$N^D_t(j) = \left(\frac{W_t(j)}{W_{J,t}}\right)^{-\eta_J} N^D_{J,t}.$$

The wage-setting behavior of resident households is unchanged compared to EAGLE. Because LU-EAGLE focuses on Luxembourg, the behavior of cross-border workers is not explicitly modeled and the labor market equilibrium is thus closed by the condition

$$W_{CB,t} = \kappa_{CB} W_{R,t},$$

with $\kappa_{CB} \in [0,1]$.

C.3. **Transfers.** A fraction $\theta_{CB} \in (0,1)$ of government transfers in the Luxembourg block is paid to cross-border households. Hence, the splitting of domestic transfers between I-type and J-type households becomes

$$(1-\theta_{CB})TR_t = (1-\omega)TR_{I,t} + \omega TR_{J,t}.$$

C.4. **Net foreign asset positions.** In the Luxembourg block, the net foreign asset position equation becomes

$$S_t^{LU,US}\left(R_t^{US}\right)^{-1}B_t^{LU*}+S_t^{LU,REA}\left(R_t^{REA}\right)^{-1}B_t^{LU^{REA}}=S_t^{LU,US}B_{t-1}^{LU*}+S_t^{LU,REA}B_{t-1}^{LU^{REA}}$$
$$+TB_t^{LU}-\left(1-\tau_t^{N,LU}-\tau_t^{W_h,LU}\right)W_{CB,t}^{LU}N_{CB,t}^{D,LU}-\theta_{CB}TR_t^{LU}.$$

The left-hand side of the equation shows the current net foreign asset positions in U.S. and euro bonds. On the right-hand side, the three first terms correspond to income from international assets and to the trade balance. The last two terms correspond to the (net of taxes) wage and transfer payments made to the rest-of-euro-area block.

Symmetrically, in the rest-of-euro-area block the equation is given by

$$S_t^{REA,US}\left(R_t^{US}\right)^{-1}B_t^{REA*}+\left(R_t^{REA}\right)^{-1}B_t^{REA^{REA}}=S_t^{REA,US}B_{t-1}^{REA*}+B_{t-1}^{REA^{REA}}$$
$$+TB_t^{REA}+S_t^{REA,LU}\frac{s^{LU}}{s^{REA}}\left[\left(1-\tau_t^{N,LU}-\tau_t^{W_h,LU}\right)W_{CB,t}^{LU}N_{CB,t}^{D,LU}+\theta_{CB}TR_t^{LU}\right].$$

Remark the presence of the relative size factor s^{LU}/s^{REA}, required to maintain proper scaling of the variables between the different blocks. The additional income represented by these wage and transfer payments also shows up on the income side of households' budget constraints in the rest-of-euro-area block.

BANQUE CENTRALE DU LUXEMBOURG
EUROSYSTÈME

2, boulevard Royal
L-2983 Luxembourg

Tél.: +352 4774-1
Fax: +352 4774 4910

www.bcl.lu • info@bcl.lu