72
Economie et Statistiques
Working papers du STATEC

Janvier 2014

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I wish to thank Serge Allegrezza and Guy Schuller for asking me to work on this project, and for their suggestions and comments. Special thanks are due to Ferdy Adam for his constant supervision, advice and support. I also thank seminar participants at STATEC, members of the Luxembourgish Comité de Prévision and of the EU Output Gap Working Group for comments and discussion. Last but not least, I wish to thank Yulyia Rychalovska, Charles-Henri diMaria, Lionel Fontagné, Massimiliano Marcellino, Anne Dubrocard and my colleagues at the EPR2 Unit.

The EU Commission production function approach to estimate output gap: the case of Luxembourg

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Abstract

The Fiscal Compact obliges member states to limit net borrowing. Public budgets, however, are allowed to vary with the state of the economy, and deficits may be recorded during a recession, leading to the concept of cyclically adjusted balance (CAB). This study describes the production function approach adopted by the EU Commission to compute potential output and the CAB for EU member states, and discusses its application to the case of Luxembourg. The empirical framework established by the Commission is applied directly to national accounts data from STATEC and results compared with the available estimates released by the Commission. Several methodological differences lead to slightly more optimistic dynamics of productivity and potential growth, and show the limits of the "one-size-fits-all" EU approach.

KEYWORDS: Fiscal compact; output gap; potential output; state-space models; Kalman filter.

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The new Treaty on Financial Stability, Coordination and Governance in the Economic and Monetary Union, also known as the Fiscal Compact, obliges member states to limit net borrowing. Public budgets, however, are allowed to vary with the state of the economy, and deficits may be recorded during a recession. This poses the problem of identifying correctly economic cycles, and makes structural economic variables such as potential output and the NAIRU — the non-accelerating inflation rate of unemployment — essential tools for evaluating fiscal stances in European countries. Such variables are used to assess changes in public budget balances, to identify underlying structural deficits and the impact of cyclical factors on budgets. Structural variables, however, are not observable, which makes their measurement difficult and their adoption for policy making problematic.

The concept of output gap, defined as the deviation of observed output from potential output, identifies cylical economic fluctuations. Its measurement depends on the availability of a measure of potential output, a theoretical concept that indicates the level of aggregate activity of an efficient economy that fully uses all factors of production. In practise, economists do not observe potential and have to identify it from the observed data using a theoretical model or ad-hoc assumptions on the evolution of output over time. After the 2007-2009 recession, the economic policy debate has often focused on the effective size of the gap. Some economists have argued that the gap is low, because the crisis' structural effects on western economies have permanently lowered potential growth. Others have countered that the gap is large, and economies are operating far below capacity. This controversy shows the difficulties posed by the measurement of a structural measure of economic activity. The estimation of potential, however, is crucial in the new policy framework set by the Fiscal Compact.

In this context, the Output Gap Working Group, set up by the European Commission and member states, has elaborated an empirical method to compute potential output and the gap which uses time series filtering techniques in a production function setting. The production function framework will be used by the Commission to compute official forecasts of potential output and the gap, to assess structural deficits and to implement surveillance on the member states' fiscal policy. The final goal is to use the derived measure of output gap to compute cyclically-adjusted budget balances (CABs). The CAB is defined as the budget balance that would prevail if an economy's was operating at potential. In other words, the framework will be used to attribute budget (im)balances to either the economic cycle or to structural factors. The main idea of the Commission approach is to establish to what extent production is constrained by the available technology and factors of production and to use this information to build forecasts of structural variables. On the technical side, the EU methodology reflects recent research which models stochastic trends in output and, at the same time, aims at capturing economic fundamentals. In addition, the need of modeling potential output as a smooth trend, and to obtain measures of potential output that work well both in the present and near future, motivate the choice of techniques made by the Commission. The main influences on the EU methodology are those of the studies by Giorno et al. (1995) and Kuttner (1994), which are reviewed in the following section.

This report decribes the production function approach adopted by the EU Commission to assess output trend and fluctuations for EU member states, and discusses its application to the case of Luxembourg. The empirical framework established by the Commission is applied directly to national accounts data from STATEC and results compared with the latest available estimates released by the Commission. Section 1 briefly reviews the key concepts of the EU approach and outlines the evolution in the estimation methodology of potential and the gap in use by economists, institutions and policy makers. Section 2 outlines the EU methodology to estimate the output gap. The remaining of the report discusses the application of the EU methodology to Luxembourg, by comparing data and results available at STATEC to those published by the Commission. Section 3 compares the data sources. Then, section 4 presents measures of trend TFP, potential output and the gap obtained by applying the production function methodology to national accounts data sourced from STATEC, and compares such measures to those presented by the Commission in the Spring 2013 forecasting exercise.

1 The measurement of output gap and potential output in the economic literature

Potential output and the output gap are essential concepts for economic policy, as they capture two different economic forces: potential output accounts for long-term economic growth, while the gap captures temporary fluctuations such as booms and recessions. Economic growth is usually explained by the dynamics of variables such as population, capital accumulation and factors' productivity, whereas fluctuations in the short-term are linked to shifts in demand, and monetary and exogenous shocks. This conceptual dicothomy justifies the empirical decomposition of real GDP, which measures aggregate output, into a trend and a cycle component that account, respectively, for long-term changes and short-term fluctuations around the trend. To do so, economists have elaborated different methods but the question of the measurement of potential growth and the gap is still far from being settled. What follows recalls briefly the main views of potential output stemming from economic theory, and gives a concise review of the empirical methods for the measurement of output gap that are relevant to the Commission's approach.

Potential output and the gap in the theory

The economic theory offers two main views of potential output. The first is the long-run concept of growth models (Solow, 1956; Romer, 1991), where potential output is driven by technological changes. The second is the idea of equilibrium output implied by business cycle or new keynesian models, which focuses on short-run dynamics. While in business cycle models the equilibrium output fluctuates in the short-term in an optimal manner, new-keynesian models highlight the role of sticky prices/wages in determining short-term economic fluctuations. Output gaps are associated to changes in inflation, so that potential output is the level of output attained in absence of inflationary pressures. The failure to observe very low inflation or even deflation has questioned the theoretical link between inflation and output gaps. Meier (2010) examines the dynamics of inflation by focusing on periods of persistent large output gaps. This author does not propose new methods to estimate the gap but focuses rather on its link with inflation; his work, however, is relevant to this study because, as it will be discussed later, the relation between inflation and structural variables is used by the Commission to identify one of the key ingredients of the production function model, namely the NAWRU. Meier's study confirms the link between the gap and slowing inflation. The link between output gaps and inflation is also at the center of the recent political economy debate. Paul Krugman argues that the reason for failing to observe deflation lies in nominal rigidities characterising advanced economies.[1] In an interesting review, Basu and Fernald

[1] One can see `http://krugman.blogs.nytimes.com/2013/03/05/why-dont-we-have-deflation/`.

(2009) discuss the usefulness of the different concepts of potential output and gap for policy making, and note that potential output is far from being a smooth process in the short-run.

Empirically, the main difficulty in the estimation of output gap consists in the fact that potential output is unobservable. The empirical literature offers two main approaches to the measurement of the output gap: 1) the *time-series* approach, based on the univariate time-series properties of the output series; 2) an alternative approach based on the concept of *production function*.

The measurement of potential output: the time series approach

The time-series approach models the output series as a univariate stochastic process, made up by a trend and a cycle component. Output y at time t can be written as:

$$y_t = y_t^* + c_t \tag{1}$$

The trend component y^* represents the equilibrium or potential output whereas the cycle c captures economic fluctuations. (For this reason, in what follows the term "trend output" is equivalent to the term "potential output".) The problem with equation 1 is that the variables y^* and c are not observed, only y is. As a result, methods need to be devised to estimate the unobservable variables exploiting the information contained in the observed data. In practice, methods differ in the choice of the estimation ("filtering") technique that allows to separate the short from the long-run component of output, and in the inclusion of economic information in the model through multivariate extensions of univariate filters.

A solution to the problem exposed above is to write a regression-like model, and to use the data to estimate the trend in output while the cyclical component is defined as a residual. The most simple version of this approach is the linear trend model:

$$y_t = \alpha_0 + \alpha_1 t + e_t \tag{2}$$

(Here, y and t denote, respectively, output and a time trend; the αs are parameters to be estimated; e is an *iid* error term.) One problem with this model is that changes in the linear trend are not allowed. A version of the linear trend model which allows changes in the trend is the split-time trend model:

$$y_t = \alpha_0 + \sum_i \alpha_i t_i + e_t;$$

Another model that also allows changes in the trend is the Hodrick-Prescott filter (hereafter HP) (Hodrick and Prescott, 1997), which has become widely popular in policy making. In the HP method, the trend component of output, y^*, is chosen to minimise the following objective function

$$\sum_t (y_{t+1} - y_{t+1}^*)^2 + \lambda \sum_t [(y_{t+1}^* - y_t^*) - (y_t^* - y_{t-1}^*)]^2; \tag{3}$$

Here, λ is a pre-set smoothness parameter that assigns a penalty to the variability in the trend component.[2]

[2] The minimisation problem of equation 3 can be seen as a bias (deviation from trend) - variance (smoothness) trade-off, which is controlled by the parameter λ. The choice of λ is crucial in assigning fluctuations to either the cyclical or trend component of the model. Ravn and Uhlig (2002) argued that lambda should vary with the frequency of the observations. In practise, the λ parameter, which controls the smoothness of the series, is usually set by the researchers based on prior assumptions on the acceptable degree of trend volatility.

Several studies have pointed out the drawbacks of the HP filter, and questioned its suitability for policy making. The HP filter suffers of the so-called end-point problem, that is, at the end of the sample period the filter gives trend estimates too close to the observed data.[3] Another problem is that the filter does not adjust to the properties of the time series being studied. Thus, when the underlying model is not correct, the filter does not attribute cyclical movements correctly (this was demonstrated by Cogley and Nason, 1995). [4] A practical consequence of such drawbacks, the HP filter excludes prolonged deviations of actual output from potential output. Hence, persistent (long-lasting) slumps are interpreted as a permanent decline in potential output.[5]

In a seminal contributions, Nelson and Plosser (1982) showed that stochastic trends are the source of non-stationarity of key macroeconomic time series such as GDP and prices; these authors argue that variations in observed output are originated both by the cyclical stationary components and by the non-stationary stochastic component; thus, "the empirical analyses of business cycles based on residuals from fitted trends lines are likely to confound the two sources of variation, greatly overstating the magnitude and duration of the cyclical component and understating the importance of the growth component" (Nelson and Plosser, 1982, p. 160).[6] Clearly Nelson and Plosser (1982) findings question the suitability of the models above, all based on linear (piecewise) models for the trend, as well as the HP filter based on the local linear trend model.

The limitations of the HP filter, and the need of taking into account the presence stochastic trends in economic time series have motivated the subsequent research on uncovering trends in output and changes in the methodology adopted by policy-making institutions.

The univariate time series approach illustrated above has been extended to include information from economic relations, such as Okun's law and Phillips curves. An example is the model adopted by the Bank of Canada for policy making, which assumes that inflation is a function of the cyclical component of output (St-Amant and van Norden, 1997). The information from a Phillip's curve is included into a HP filter as the squared error of the fitted

[3] This issue has been examined by many authors. It is not possible for reasons of space to give an exhaustive review of this literature. One can see Baxter and King (1999) and Cayen and van Norden (2002).

[4] This is because the HP filter is equivalent to a local-linear trend model, which implies that variables are I(2) processes (Harvey and Jager, 1993). Many important economic variables, however, are usually modelled as I(1) processes. See also footnote number 6.

[5] A further problem is that the HP filter assigns fluctuations either to the cycle or to the trend component of a series, without allowing for any other type of randomness (ie outliers) (Mohr, 2005).

[6] The term persistency refers to the memory feature of a time series, and, in particular, to the impact of random shocks on future values of the series. Different time series processes deliver different implications in terms of the effects of shocks. If data are modelled as stochastic trends, random shocks have a permanent effect on variables. In other words, current shocks on GDP will affect the long-run level of the variable. To see this formally, recall that a stochastic-trend process can be modelled as:

$$y_t = y_{t-1} + e_t;$$

where y is a macro variable and e an *iid* error term ($e \sim (0, \sigma^2)$); y at a future date T can be expressed as the sum of past shocks:

$$y_T = y_0 + \sum_{i=1}^{T-1} e_{T-i}$$

(y_0 is the initial value). Stochastic trend models are also referred to as unit-root, or I(1) processes. This terminology is motivated by the fact that the first difference of the variable y is a stationary process: indeed, $\Delta y_t = y_t - y_{t-1} = e_t$, and e is stationary by definition. (Note that the term I(2) denotes a variable that needs to be differentiated twice to be found stationary. If this was the case, $\Delta^2 y_t = e_t$.)

inflation function. St-Amant and van Norden (1997) also consider multi-variate extensions of the HP filter and examine the performance of dynamic multivariate models such as VAR methods, concluding that the uncertainty related to these methods is high. A family of VAR models, the structural VARs (SVARs), also offer means of decomposing output in transitory and long-term components combining multi-variate time series techniques with information from economic theory.

Another type of models, usually referred to as unobserved component models, specify dynamic processes for both cycle and trend components and estimate them jointly. An influential study is the one of Kuttner (1994), which combines economic information and dynamic specification. According to Kuttner, any method to estimate potential output should have the following key characteristics: 1) it should be able to detect changes in trend in a timely manner; 2) and to produce not only output gap series but also measures of its (time varying) uncertainty. This author proposes a time-series model of the evolution of output complemented by a version of the Phillips curve, which models the relation between inflation and the cyclical component of output. The empirical system of equations is as follows:

$$y_t = y_t^* + z_t \tag{4}$$
$$z_t = \alpha_1 z_{t-1} + \alpha_2 z_{t-2} + e_t \tag{5}$$
$$\Delta y_t^* = \mu + u_t \tag{6}$$
$$\Delta\pi_t = \mu_\pi + \lambda_1 \Delta y_{t-1} + \lambda_2 \Delta z_{t-1} + v_t \tag{7}$$

The first equation is an identity that defines output gap: observed output (y) is the sum of a trend (y^*) and cycle (z) component. The second equation tells that the cyclical component z of output follows an autoregressive process of order 2 (AR(2)).[7] The third equation models the trend component of output y^* as a stochastic trend process.[8] (e and u are random shocks.) The last equation links changes in inflation ($\Delta\pi$) to output growth, the gap and an *iid* error term v, and is interpreted by the author as "an aggregate dynamic supply relationship involving the gap". (All other terms in the model — αs, μ, μ_π, λs — are parameters to be estimated.) Clearly Kuttner's model implies that real GDP follows a stochastic trend process, as it is expressed as the sum of a random walk and a stationary process (eq. 4).

The model above involves unobserved variables, which make standard econometric techniques unsuitable for its estimation. Statistical techniques, however, have been elaborated to solve the joint problems of parameters estimation and recovering of unobservable variables: these consist in writing the model using a state-space representation and applying a Kalman-filter procedure to estimate unobserved components (Hamilton, 1994a). The state-space representation of Kuttner's model is given in Planas et al. (2008). These techniques are briefly reviewed in the appendix C to this report.

A similar model to Kuttner is used by Planas et al. (2008), who revisit the measurement of output gap, and Planas et al. (2007) to study the effect of taxes on unemployment.

[7]The general formulation of an autoregressive (AR) process for a variable y is

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + ... + \alpha_p y_{t-p} + \epsilon_t$$

where ϵ is an *iid* error term and the αs are the so-called auto-regressive parameters; p is known as the order of the process, and indicates how many lags (past values) of the variable y enter the dynamic specification. As opposed to random walks, AR processes are stationary and the effects of shocks fade away over time.

[8]$\Delta y_t^* = \mu + u_t$ can be re-written as $y_t^* = \mu + y_{t-1}^* + u_t$. This stochastic process is commonly referred to as a random walk with drift (the drift term is the parameter μ).

The recent financial and sovereign debt crisis have sparked a renewed interest in the measurement of output gap and the usefulness of the HP filter. Most recent criticism has focused on the instability of the filtering techniques in real-time. An important contribution is the article of Orphanides and van Norden (2002). These authors analyse the performance of several models, such as the linear trend, the HP filter and Kuttner's, focusing on the effects of data revisions and the added information provided by the availability of longer time series. This added information, and the consequent parameters' instability and revisions of estimates, is identified as the source of the unreliability of available output gap series.[9]

In summary, the time series approach views **potential output** as the level to which output reverts when the effect of transitory shocks dissipates. Thus, the **output gap**, related to the transitory component (the cyclical movement) of output, measures the difference between such level and observed output.

The production function approach

An alternative concept of potential output is related to the notion of production function, which links output to total factor productivity (TFP) and to aggregate inputs, capital and labour. In this context, potential output is the level of output attained in correspondence of the "normal use" of factors to production. In other words, potential output is determined by the current technology, or the current technical ability to produce. The approach relies on the availability and reliability of measures of potential TFP, potential labour (L) and potential capital stock (K), and utilisation rates. It also requires to estimate an aggregate production function. Ideally, one would resort to frontier techniques to estimate potential output and deviations from it.[10] The production function approach is applied by assuming a Cobb-Douglas functional form for aggregate production, and potential output is recovered by plugging-in (filtered) inputs to production.

The Commission implements a version of this latter approach, first proposed at the OECD by Giorno et al. (1995). (The study of Giorno et al., 1995, compares the performance of the production function method to the one of HP and time trends for OECD countries, and discusses implications for budget balances and fiscal policies.) A similar method is also in use at the IMF.[11] The following section is devoted to a more detailed explanation of the production function approach.

2 The EU model: the production function method

This section reviews and discusses the production function approach used by the European Commission to compute output gap and potential output as described in D'Auria et al. (2010). According to this approach, a production function links the level of aggregate economic activity (output) to two inputs to production, namely capital and labour. A residual — the part of output which is not accounted for by the contributions of the inputs — reflects efficiency trends and capacity utilisation and is interpreted as total factor productivity

[9] "The bulk of the problem is due to the pervasive unreliability of end-of-sample estimates of the output trend" (Orphanides and van Norden, 2002, p. 582).

[10] Frontier techniques are either parametric (stochastic frontier approach - SFA) or non-parametric. The latter method is so-called because it does not assume a functional form for the produciton function. An example of an application of a deterministic non-parametric frontier approach is given in Peroni (2012).

[11] One can see, for example, an application to Poland in Epstein and Macchiarelli (2010).

(hereafter TFP). A two-factor Cobb-Douglas production function is specified for each country for a given value of the labour share on income. Then, estimated residuals from these equations are smoothed to give trend TFP. Finally, potential output is obtained by "plugging-in"' the production function the actual capital stock, trend TFP and an estimate of aggregate potential employment.

Two crucial features of this approach allow the Commission to construct measures of the potential inputs: 1) potential employment corresponds to the use of labour forces consistent with non-accelerating inflation; 2) observed TFP is linked to cyclical indicators of capacity utilisation.

2.1 The specification of the production function and potential output

Generally speaking, a production function is a relation between inputs to production and output. The EU model is based on the two factors Cobb-Douglas production function:

$$Y = TFP * L^{\alpha} * K^{1-\alpha}; \tag{8}$$

where TFP denotes the so-called Solow Residual, K and L are capital and labour inputs, and α is the marginal productivity of labour. D'Auria et al. (2010) rewrite the production function as follows:

$$Y = (U_L E_L L)^{\alpha} (U_K E_K K)^{1-\alpha} = L^{\alpha} K^{1-\alpha} \underbrace{\left(U_L^{\alpha} E_L^{\alpha} U_K^{1-\alpha} E_K^{1-\alpha}\right)}_{TFP}; \tag{9}$$

Here, U and E represent, respectively, the degree of capacity utilisation and the level of efficiency in the use of each input to production. Equation 9 differs from the standard formulation of equation 8 in that the variables E and U provide an explicit link between production and trend-cycle components. For example, U_k represents the fraction of an economy's stock of capital that is actually used in the production process; U_L refers instead to the use of labour capacity and depends on factors such as labour market participation trends and unemployment. Clearly these two variables are affected by the economic cycle, and one would expect them to be higher in expansions and lower in recessions.

The model above makes the following assumptions:

1. The functional form of the production function is specified as a Cobb-Douglas;
2. Returns to scale are constant (the sum of the exponents of the inputs to production is equal to 1);
3. Markets clear (perfect competition);

These assumptions allow to compute the parameters α and $1 - \alpha$ using historical data on wages, avoiding difficulties related to the direct measurement of capital remuneration. In practise, under perfect competition, the parameter α can be estimated by dividing total workers' remuneration by total income, as measured by GDP. [12] The value assigned to the parameter α is set by the Commission to 0.65 and corresponds to the historical average of wage share data for the 27 member countries.

[12] Under perfect competition, the marginal productivity of labour equals the wage rate.

In this framework, potential output is defined as the level of output which corresponds to a full use of inputs. The idea is that the same relation that describes the determination of current output must apply also for output at potential. The Commission, based on equation 9, writes potential output as follows:

$$Y^p = \underbrace{(E_L^{T\alpha} E_K^{T1-\alpha})}_{trend\ TFP} L^{p\alpha} K^{1-\alpha}; \tag{10}$$

Here, the potential Solow residual, or trend TFP, is interpreted as the product of labour and capital trend efficiency. $E_{L,K}^T$ represent trend efficiency ("a normal level of efficiency of factor inputs"). Note that the maximum contribution of capital stock to potential output is the full use of the existing capital stock in an economy, so that $U_K K = K$ at potential. This equation implies that, in order to calculate potential output, one needs to obtain trend TFP and the labour potential L^p. In the Commission methodology, each component of the equation is estimated separately. The estimation strategy for trend TFP and labour potential are outlined in the following sections.

Finally, the output gap is derived from the comparison of observed to potential output:

$$Y_{gap} = \frac{Y - Y^p}{Y^p} * 100. \tag{11}$$

2.2 The estimation strategy for trend TFP

One of the crucial implications of the Cobb-Douglas framework outlined in the previous section is that current TFP reflects both cyclical and trend components. While the latter are unobserved, the TFP cyclical component depends on current economic conditions, so it must be linked to observable variables. D'Auria et al. (2010) argue that capacity utilisation measures are good candidates for such observables: data are available for European economies and series are found to be highly correlated with current TFP. This allows the authors to estimate a bivariate model of trend TFP with unobserved components inspired by Kuttner's model (1994).

Recall from equation 9 that "observed" TFP is related to labour and capital efficiency and to inputs' capacity utilisation, as follows:

$$TFP = U_L^{\alpha} E_L^{\alpha} U_K^{1-\alpha} E_K^{1-\alpha}, \tag{12}$$

where U denotes capacity utilisation and E denotes efficiency. Taking the logs of both side of the equation, which permits linearisation, one gets:

$$log(TFP) = \alpha(log(U_L) + log(E_L)) + (1-\alpha)(log(U_K) + log(E_K)); \tag{13}$$

Rearranging the equation above and renaming the variables so that lowercase names denote logarithms, one can rewrite log TFP as the sum of two unobservable components, the cycle (c) and the trend (p) as follows:

$$tfp = \alpha(u_L + e_L) + (1-\alpha)(u_K + e_K) = \underbrace{\alpha u_L + (1-\alpha)u_K}_{c} + \underbrace{\alpha e_L + (1-\alpha)e_K}_{p} \tag{14}$$

One can see that the cycle is assumed to be linked to capacity utilisation whereas the trend is linked to efficiency in inputs' use.

The empirical model for the construction of trend TFP exploits equation 14 and the assumed link between observed variables and the cyclical components of TFP. The observable variable linked to the cyclical component of TFP is a structural composite indicator of capacity utilisation. The model, which links four variables, *tfp*, u, c, and p, and specifies time series dynamics for trend and cyclical component of TFP, is composed by the following equations:

$$tfp_t = p_t + c_t \tag{15}$$
$$u_t = \mu_U + \beta c_t + e_{Ut}, \qquad \beta > 1 \tag{16}$$
$$\Delta p_t = \mu_{t-1} \tag{17}$$
$$\mu_t = \omega(1-\rho) + \rho\mu_{t-1} + a_{\mu t} \tag{18}$$
$$c_t = 2Acos(2\pi/\tau)c_{t-1} - A^2 c_{t-2} + a_{ct}; \tag{19}$$

The first equation of the model simply rewrites equation 14. Equation 16 represents a regression of the cyclical indicator of capacity u, on the intercept μ_U and the (unobserved) cyclical component of tfp, c; e_U is an error term. The key parameter to be estimated is β, because it measures the strength of the link between the cyclical indicator, denoted by u and the TFP. One difficulty with this approach is that c is made up of cyclical components related respectively to capital (u_K) and labour (u_L), which cannot be distinguished in the equations above. The capacity utilisation indicator, however, is expected to be strongly correlated to capital utilisation and, to a lesser extent, to labour utilisation.[13]

Equations 17 and 18 tell that the trend component of TFP, p, follows a random walk with drift. The drift, denoted by μ, is itself random and follows an AR process, where ρ is the autoregressive parameter. (These type of models, called *damped trend* models, differ from standard random walks as the drift is itself a random process rather than a parameter.) The cyclical component c follows an AR process of order 2 with cyclical parameters A and τ, where A gives the amplitude of the cycle and τ its periodicity. a_μ and a_c are *iid* error terms.

As noted in the previous section, models such as those of equations 15–19 cannot be estimated with standard econometric methods as they involve unobservable variables. When some variables of interest are unobservable, econometricians resort to state-space models. These models allow them to "reconstruct" from the data the unobserved series of interest using a procedure called Kalman filtering. The model's parameters are usually estimated using Maximum-Likelihood methods (Hamilton, 1994b,a). A state-space model is composed by an observational part, which uses identities and structural relations among variables, and a measurement part, which describes the variables (also called states) evolution over time. Here, equations 15 and 16 can be regarded as the observational part of a state-space model,

[13] Equation 16 is obtained exploiting the correlation between u_l and u_k, respectively labour and capital component of the cyclical variable, specified as $u_l = \gamma u_k + \epsilon$. Consider again equation 14 and focus on the cyclical part. It works out as follows:

$$\begin{aligned} c &= \alpha u_l + (1-\alpha)u_k = \alpha(\gamma u_k + \epsilon) + (1-\alpha)u_k \\ &= (\alpha\gamma + 1 - \alpha)u_k + \alpha\epsilon, \qquad \text{which gives} \\ u_k &= \frac{1}{1-\alpha(1-\gamma)}c + e \end{aligned}$$

Thus, equation 16 is interpreted as the link between unobservable cyclical component of log TFP and capital utilisation, with the parameter β reflecting labour share (α) and the correlation between labour and capital utilisation (γ).

where p and c are the unobserved state variables. This structure is inspired by Kuttner (1994) (reviewed in section 1), who associated a standard observation equation to a regression with unobserved quantities containing economic information (a version of the Phillips curve) with the objective of modelling potential output.

The Commission model estimates jointly the unobserved variables and parameters using a Bayesian procedure which simulates joint posterior distributions from the data likelihood and prior distributions. More details on model estimation are given in section 4.3 and in the Appendix C to this report.

For Luxembourg, the estimated β of equation 16 is equal to **1.275** (increased to 1.4 in the spring 2013 exercise), which signals a strong link between TFP cycle and capacity utilisation indicator (see D'Auria et al., 2010, p. 21). Other member countries have similar or higher values, whereas only Greece, Netherlands and Portugal have point estimates lower than 1 (one should note, however, that confidence intervals are large). The first two columns of table 1 below report a list of parameters and their estimated values according to estimates released by the Commission in spring 2013. Figure 1 presents the estimates of growth rates of trend TFP against observed values published by the Commission.

The Commission argues that this method produces smooth trends for TFP immune from the end-of-period bias which affect the HP filtering technique. It is noted that the high variability in TFP series is not reproduced by the trend component, and the methodology seems able to accurately capture trend and cycle, by correctly attributing large falls in TFP occurred in correspondence of the two recessions of 2001-2003 and 2007-2009 to the cyclical component.

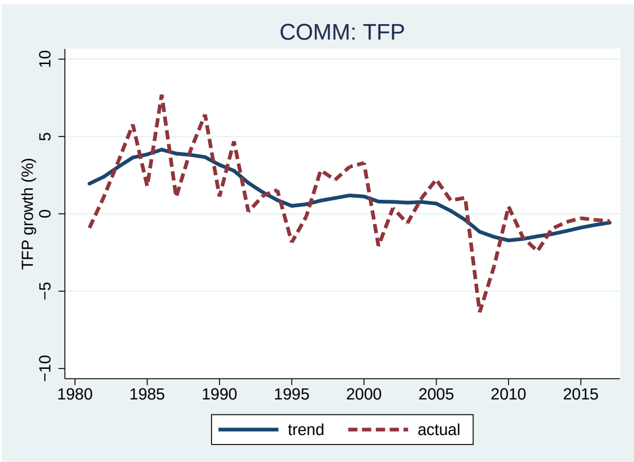

Figure 1: **Trend and observed TFP growth** (% annual change). (Source: EU Commission.)

Table 1: **Commission: estimation results for TFP and Nawru models for Luxembourg**

TFP model		NAWRU model	
Parameters	Estimates	Parameters	Estimates
Equation with observables:			
β	1.40	β_N	0.63 (-1.96)
Time series equations cycle:			
τ	8	ϕ_1	1.03 (5.63)
A	0.42	ϕ_2	-0.45 (-2.64)
$V(a_c)$	0.001	$V(a^c)$	0.12
trend:			
ω	0.015		
ρ	0.80		
$V(a_\mu)$	0.000005	$V(a^*)$	0.03
		$V(a^\lambda)$	0.0014

Legend: Significance statistics for ML estimates of parameters are reported in parentheses when available. V denotes the estimated variance of the error terms. Source: European Commission https://circabc.europa.eu/.

2.3 The estimation of the NAWRU and potential labour input

The Commission computes potential labour as follows:

$$L^P = \underbrace{(\underbrace{POPW * PART^S}_{labour \quad force} *(1 - NAWRU))}_{potential \quad employment} *HOURS^S \tag{20}$$

Here, POPW is population of working age, $PART^S$ is a smoothed participation rate, NAWRU is structural unemployment, and $HOURS^S$ is trend average hours worked. (The suffix S denotes variables which are smoothed using a HP filter.) NAWRU is the (long-term or structural) rate of unemployment which is consistent with non-accelerating wage inflation.[14] The first part of this expression gives the trend labour force, which, multiplied by the NAWRU term, gives total potential employment. Potential employment multiplied by the trend number of hours gives the total potential labour input.

Structural unemployment, the NAWRU, is unobservable. The series is constructed resorting to an unobserved component model which includes a Phillips curve and specifies dynamic

[14]NAWRU stands for Non-Accelerating Wage Rate of Unemployment. The other commonly used measure of long-term unemployment, NAIRU, is instead consistent with non-accelerating price inflation.

processes for trend and cyclical unemployment:

$$\Delta^2 w_t = \phi_{prod}\Delta^2 prod_t + \phi_{ws}\Delta^2 ws_t + \phi_{tot}\Delta^2 tot_t - \beta_N(u_t - u_t^*) + v_t \tag{21}$$

$$u_t = u_t^* + u_t^c \tag{22}$$

$$\Delta u_t^* = \lambda_{t-1} + a_t^* \tag{23}$$

$$\Delta\lambda_t = a_t^\lambda \tag{24}$$

$$u_t^c = \phi_1 u_{t-1}^c + \phi_2 u_{t-2}^c + a_t^c \tag{25}$$

Here, w denotes the nominal wage level, $prod$ labour productivity, ws wage share on total income, tot terms of trade;[15] u and u^* denote, respectively, the observed unemployment rate and the long-run equilibrium unemployment (or NAWRU); v is an iid error term. β_N and the ϕs are parameters estimated via Maximum-Likelihood. Equation 22 is an identity and represents unemployment as the sum of a cyclical and unobserved part, denoted respectively by u_c and u^*. Equations 23–25 constitute the measurement part of the model and specify the dynamics of the unobserved components. Cyclical unemployment follows, as for TFP, an AR(2) process with autoregressive parameters ϕs. Equilibrium unemployment (the NAWRU) follows a second order random walk;[16] a^*, a^λ, a^c are *iid* random terms.

Equation 22 above is a version of a Phillips curve, derived from a model of the labour market detailed in Planas et al. (2007), inspired in turn by the model of Blanchard and Katz (1999). The curve describes the dynamic adjustment of wages to economic conditions; it tells that short-term increases in nominal wage inflation are associated to a decrease in the unemployment gap (the difference between observed unemployment rate and the NAWRU); viceversa, downward pressures on nominal wages are associated to increases in short-term unemployment (relatively to the NAWRU). Thus, β_N is a key parameter which determines the magnitude of the adjustment of wage inflation to the unemployment gap.[17] For Luxembourg, the Commission reports the following estimates:

$$\begin{aligned} \Delta^2 w_t &= 0.28\Delta^2 prod_t + 0.03\Delta^2 tot_{t-1} - 0.93(u_t - u_t^*) && (26)\\ & \quad\ (2.75) \qquad\qquad (0.22) \qquad\qquad (2.83) && (27) \end{aligned}$$

One can see that the coefficient on the unemployment gap (the value of β_N) is significant, which supports the model specification. (The term in parentheses are t-ratios.) The effect of changes in productivity on changes in wage inflation is also significant.

Figure 2 plots the estimated NAWRU along with the actual (harmonised) unemployment rate up to 2017. The computations show a considerable increase in Luxembourg's NAWRU over the last decade, from 3.6 per cent in 2000 to 2006 to 4.7 in 2007 up to 5.9 in 2011. One also observes that the NAWRU estimates attribute most of the recent increase in unemployment to cyclical fluctuations, and, as a result, the NAWRU level has increased considerably. Estimates of the model for the EU15 group of countries evidence considerable cross-country variation, attributed to labour supply factors (D'Auria et al., 2010, p. 33 and following).

[15] The terms of trade are computed as the log difference between the consumer price deflator and the GDP deflator.

[16] The drift, λ, is a random walk specified by equation 24, implying that the NAWRU is a I(2) variable.

[17] The term $\Delta^2 w_t$ denotes changes in wage inflation, that is, $\Delta^2 w_t = \Delta(w_t - w_{t-1})$. All variables are in log except the unemployment rate; thus, β_N can be interpreted as an elasticity. This means that, for example for Luxembourg, a 1 percent increase in observed unemployment with respect to the NAWRU brings about a 0.9 percent decrease in wage inflation.

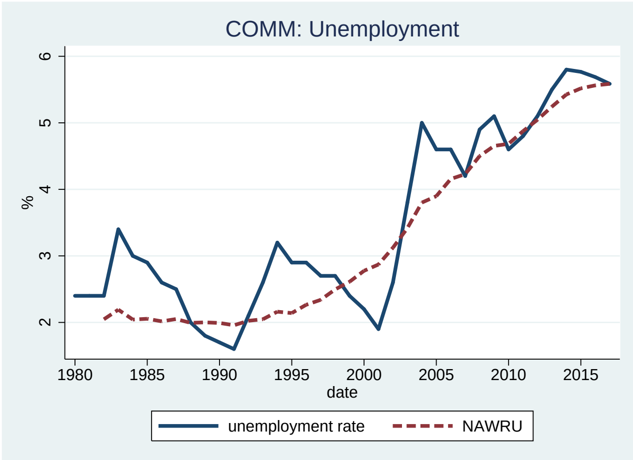

Figure 2: **Observed unemployment and NAWRU** (% on labour force). (Source: EU Commission.)

2.4 Summary

The following table summarises the Commission's spring forecasts results on potential output, the gap, contributions to growth and components of potential labour for Luxembourg. (Results presented here are those obtained with the production function method.)

Table 2: **Luxembourg: Commission results**

LU	Gap	GDP growth: actual	potential	Contributions: L	K	TFP	Labour Potential: POP	PART	NAWRU
1981									
1982									
1983	-2.8	3.0	3.3	-0.2	0.4	3.1	0.2	58.4	2.2
1984	-1.1	6.2	4.3	0.4	0.2	3.7	0.3	58.7	2.0
1985	-2.6	2.9	4.6	0.7	-0.1	3.9	0.4	59.1	2.1
1986	0.8	10.0	6.2	1.2	0.7	4.3	0.6	59.9	2.0
1987	-1.8	4.0	6.7	1.5	1.1	4.1	0.6	60.9	2.1
1988	-0.4	8.5	7.0	1.7	1.3	4.0	0.5	62.2	2.0
1989	2.2	9.8	7.0	1.8	1.3	3.9	0.6	63.6	2.0
1990	1.0	5.3	6.6	2.0	1.3	3.3	0.9	65.1	2.0
1991	3.0	8.6	6.6	1.9	1.7	2.9	1.1	66.5	2.0
1992	0.1	1.8	4.7	1.7	0.9	2.1	1.2	67.9	2.0
1993	-0.2	4.2	4.6	1.6	1.5	1.4	1.2	69.1	2.0
1994	-0.2	3.8	3.8	1.5	1.3	0.9	1.2	70.4	2.2
1995	-2.0	1.4	3.3	1.6	1.2	0.5	1.2	71.7	2.1
1996	-3.8	1.5	3.4	1.6	1.2	0.6	1.1	73.1	2.3
1997	-2.1	5.9	4.2	1.8	1.5	0.9	1.1	74.9	2.3
1998	-0.3	6.5	4.6	2.0	1.5	1.1	1.0	77.0	2.5
1999	2.3	8.4	5.7	2.3	2.1	1.2	1.1	79.4	2.6
2000	5.4	8.4	5.2	2.5	1.5	1.2	1.4	82.0	2.8
2001	2.9	2.5	5.0	2.4	1.8	0.8	1.2	84.6	2.9
2002	2.4	4.1	4.7	1.8	2.0	0.8	0.9	87.0	3.1
2003	-0.6	1.7	4.7	1.8	2.2	0.7	1.2	89.3	3.4
2004	-0.6	4.4	4.4	1.7	1.9	0.8	1.4	91.3	3.8
2005	0.1	5.3	4.4	1.9	1.9	0.7	1.5	93.1	3.9
2006	1.3	4.9	3.7	1.7	1.8	0.2	1.6	94.8	4.2
2007	4.2	6.6	3.6	1.8	2.3	-0.4	1.6	96.4	4.2
2008	1.1	-0.7	2.3	1.6	1.9	-1.2	2.0	97.6	4.5
2009	-3.9	-4.1	0.9	1.3	1.1	-1.5	2.1	98.4	4.7
2010	-1.7	2.9	0.6	1.2	1.2	-1.7	2.1	98.8	4.7
2011	-1.2	1.7	1.1	1.3	1.5	-1.7	2.6	99.1	4.9
2012	-2.0	0.3	1.1	0.9	1.7	-1.5	2.1	99.4	5.0
2013	-2.0	0.8	0.9	0.7	1.5	-1.3	1.7	99.6	5.2
2014	-1.5	1.6	1.0	0.7	1.4	-1.1	1.6	99.9	5.4
2015			1.0	0.7	1.2	-0.9	1.4	100.2	5.5
2016			1.2	0.8	1.1	-0.7	1.4	100.6	5.6
2017			1.3	0.8	1.1	-0.6	1.3	101.2	5.6

Legend: Gap is output gap calculated as percentage of potential output; actual and potential GDP (volumes) is in annual percent change; POP is population of working age, in annual percent change; PART is trend participation rate, in percent of population of working age; NAWRU is in percent of labour force. Source: European Commission spring 2013 forecasts, available at https://circabc.europa.eu/.

2.5 Discussion

The production function method described in previous sections is applied by the Commission to the data available for each EU member state. This poses the problem of how well the approach and the choice of parameters fit heterogeneous economies (in terms of economic and market structures, degrees of openness, institutions). In particular, several issues lead us to question the applicability of the production function approach to the case of Luxembourg. Firstly, the availability of data for Luxembourg is limited. For example, STATEC publishes data on GDP from 1995, due to a methodological break that applies to the series since that date. Secondly, the model of the Commission is a closed-economy one, while Luxembourg is a small very open economy. Thus, the definition of potential labour given by the Commission, and as a consequence of potential output itself, is highly problematic. This difficulty for Luxembourg stems from the large number of foreign resident workers, a component of the labour force typically characterised by high mobility, and also of a large amount of cross-border workers. Furthermore, the main motivation for the choice of a Cobb-Douglas specification is the one of simplicity. However, one should also be aware of the criticism regarding assumptions 1 to 3 (see section 2.1). In particular, the assumption of perfect competition does not seem adequate for Luxembourg industries and is not supported by the data in the studies conducted so far.[18] Moreover, the Commission assigns a value of 0.65 to the parameter α. However, historical data for Luxembourg produce an average of 0.53 for the wage share. This casts doubt on the opportunity to assign to the parameter the same value over countries characterised by different economic structures and sizes.[19]

There are also more general issues related to the employment of the Kalman filter and time series methods when time series are short. The Kalman filter is a sophisticated technique which is highly dependent on initial values postulated for the unobserved components. More in general, any forecasting techniques based on time series processes is heavily dependent on initial values. Clearly this problem is aggravated by the unavailability of long time series. If time series are very long, then the weight of the initial observation on the forecasted value of the series is negligible. (It is sufficient to look at equation 38-39 in the Appendix to see this.) The Bayesian approach used to estimate the model is also widely criticised because of its dependence on *prior* distributions, a problem aggravated by small sample estimation.

The next section discusses the data used in the empirical exercise presented in the remaining of the paper and compares them to the data used by the Commission. The final part of the article re-compute the production function using STATEC national accounts data and proposes some improvements to the Commission approach that we believe improve its applicability to Luxembourg. Differences in data and results are analysed and discussed.

[18]One can see, for example, the study by DiMaria (2008a). This author show that, using a Lerner index on national accounts data, marks-up are non-negligible across services and manufacturing industries (recall that if markets are perfectly competitive mark-ups should be equal to one). Using an approach based on cost efficiency, Peroni and Ferreira (2011) find that measures of markets' competitive pressure vary widely across Luxemourg's manufacturing industries.

[19]The issues discussed above have motivated the choice of a computational frontier approach to calculate TFP indices in the LuxKlems project (Peroni, 2012).

3 The data

This section compares the data available at STATEC to those used by the Commission to run their periodic forecasting exercises. The data will be used in the remaining of this report to estimate potential output figures for Luxembourg. The basic series are annual observations on macro variables such as GDP, capital stock, and several labour market and employment variables, sourced from the national accounts data published by STATEC. The series includes the latest official forecasts and range from 1980 to 2016.[20] The Commission uses the AMECO database, which covers the period 1960–2014 and includes macroeconomic series for EU member states, candidate and other OECD countries. (AMECO series are mainly, but not exclusively, sourced from Eurostat official statistics.)[21]

The Commission computes the labour input by multiplying the number of persons employed by hours worked.[22] The employment concept used in this calculations is domestic employment, which includes both resident and non-resident workers. At STATEC the total labour input is computed in a sligthly different manner, by multiplying domestic employment by an index of hours worked. (This gives a measure of the evolution of total working time, and will be referred to as *effective employment* hereafter.) Figure 3 shows the evolution of labour in Luxembourg in both levels and growth rates according to the different data sources. One can see that growth rates have similar patterns. During the period analysed, Luxembourg's labour input has increased steadily, with growth rates well above zero with the exception of recessive periods. (One observes the negative pick occurring in 2009 during the financial crisis.) One should also note that this pattern is due to increases in the number of employees rather than to the dynamics of hours worked, which is declining over the examined period.

Another important variable which affects both the computation of actual TFP and potential output is the capital stock. Figure 4 compares the series used at STATEC to those produced by AMECO. Here, one observes substantial differences both in levels and in the growth rates. The lower levels of capital stock recorded by AMECO are explained by the fact that that AMECO's series corresponds to net capital stock, while at STATEC gross capital stock is the preferred measure to compute TFP. Gross capital stock takes into account assets' retirement whereas net capital stock includes a measure of depreciation. The first one is usually preferred for measuring TFP as depreciation tends to make assets disappear too fast from the aggregate stock, leading to under-estimation of the capital actually used in production.[23] One should note that differences in levels persist even when comparing net capital stocks. (These are reported in figure 14 in the appendix for reasons of space.) This is due to methodological differences in the computations of the series. At STATEC, a perpetual inventory method (PIM) is applied on disaggregated data, which requires to specify retirement patterns for all type of goods. The initial capital stock is set in 1870. For a detailed description of the methodology in use at STATEC one can refer to DiMaria and Ciccone (2006). AMECO's capital stock is computed using the PIM on aggregate data, assuming that the initial capital stock (set in 1960) is set as high as 3 times the level of GDP.

[20] Forecasts are built on national account data and produced using Modux, a large scale macro-econometric model of the Luxembourg economy (Adam, 2007).

[21] The AMECO data considered here are those used to perform the spring 2013 forecasting exercise. Data were retrieved on the 13th of May 2013 on https://circabc.europa.eu/.

[22] Hours worked are annual average hours worked per employee, and are available in AMECO since 1983. Information on the source of hours worked data is not available.

[23] On this and the long debate on the appropriate capital measures to compute TFP one can see Schmalwasser and Schidlowski (2012) and Blades and zu Schlochtern (1997), as well as the OECD (2009) handbook.

Despite the differences in the levels of the series, the evolution of capital's growth rates is comparable across datasets; AMECO growth rates, however, are higher and more volatile than STATEC ones. For example, both series show a considerable slow-down in capital accumulation in 2009 and 2010. A further decrease in the capital accumulation rate also appears in the forecasting period.

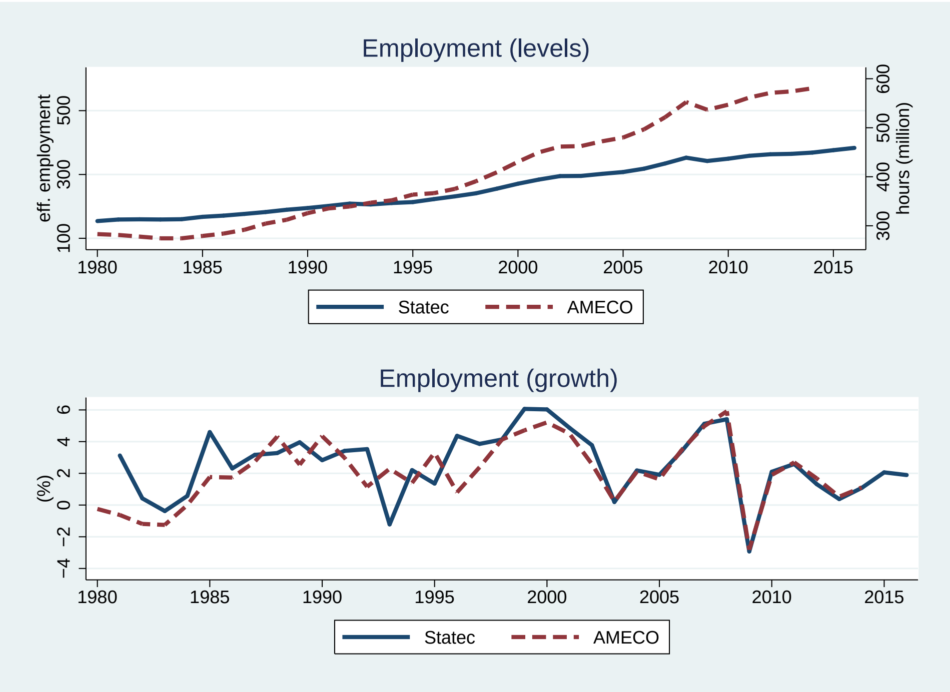

Figure 3: **Labour input 1980-2016: levels (top panel) and growth rates (bottom panel).** Growth rates are percentage annual changes. Series are effective employment (STATEC, blue line) and total hours worked (Commission, dashed red line). (Source: STATEC, AMECO.)

The other variables used in the calculations of (potential) labour input are the population series and the harmonised unemployment rate. One should note that the unemployment rate corresponds to the harmonised series and excludes non-resident workers.[24] Figure 5 plots the unemployment rates calculated at AMECO and STATEC. One observes some discrepancies in the last years of the sample, corresponding to the forecast period. STATEC data tend to produce higher figures for unemployment than those produced by AMECO. The Commission uses the population of 16-74 years of age, while at STATEC the population measure refers to the age bracket 16-64.

[24]Series available for unemployment are harmonised unemployment and registered unemployment. Harmonised unemployment is provided by the Labour Force Survey (LFS) in accordance to the ILO (International Labour Organisation). In contrast, registered unemployment is sourced by public unemployment services (ADEM for Luxembourg). There are important differences between the harmonised and the registered unemployment series. For a summary of these discrepancies one can see the interesting paper of Melis and Ludecke (2006).

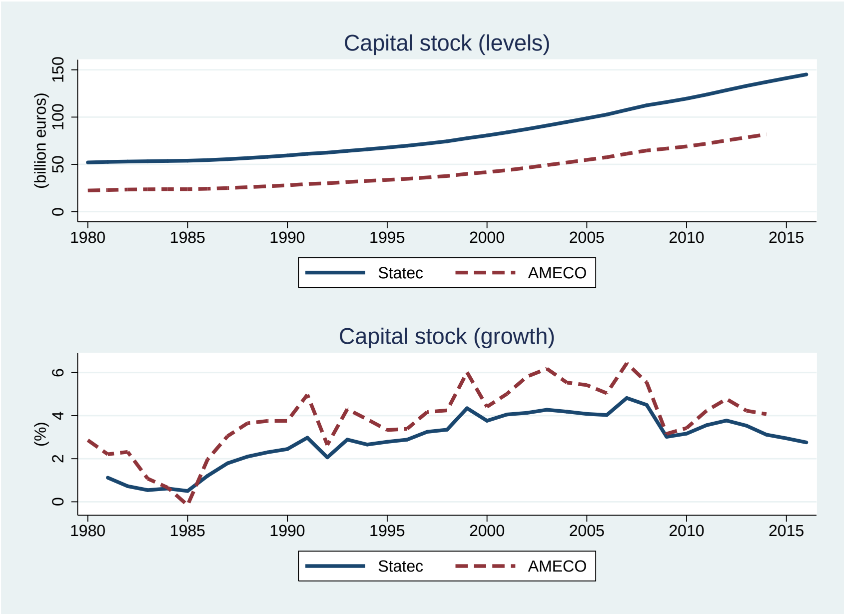

Figure 4: **Capital stock 1980-2016: levels (top panel) and growth rates.** Growth rates are percentage annual changes. STATEC (blue line) and AMECO data (dashed red line). (Source: STATEC, AMECO.)

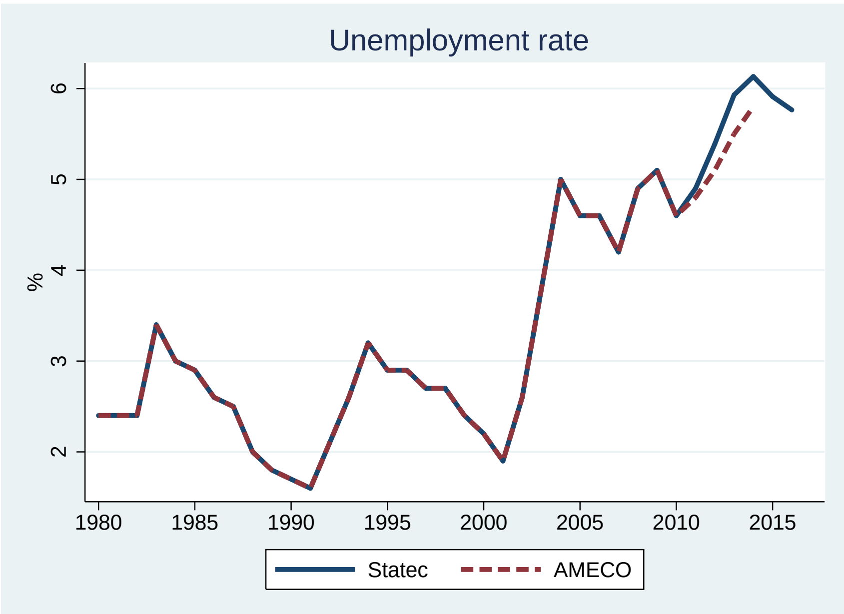

Figure 5: **Unemployment rates 1980-2016.** Data are percent on labour force. STATEC (blue line) and AMECO data (dashed red line). (Source: STATEC, AMECO.)

4 The EU methodology applied to Luxembourg data

This section presents measures of trend TFP, potential output and the gap obtained by applying the production function methodology to data sourced from STATEC, and compares results to those presented by the Commission in the forecasting exercise of Spring 2013. Figures obtained with STATEC's data will be labelled as "LUX-COM", while the results published by the Commission will be denoted as "COMM". The data used in the STATEC exercise differ in several ways from those used by the Commission, as highlighted in previous sections. In summary:

1. At STATEC, **gross capital stock** is computed using a perpetual inventory method applied on disaggregated data, while the Commission uses AMECO's net capital stock series built directly on national aggregates;

2. The **wage share** estimated using historical data for Luxembourg is equal to 0.52, while the Commission set this value to 0.65, a common value for all member states calculated as an average on historical data;

3. The Commission uses the forecasts produced by AMECO up to 2014, while the STATEC dataset includes forecasts up to 2016, so that medium-term tendencies are exogenous rather than endogenous.

A further departure from the Commission method consists in the explicit distinction between the cross-border workers and the resident workers when computing the **potential labour input series**. This is because, as already noted in this report, it is not possible to establish potential employment for cross-border workers. Thus, the potential labour input is computed as follows:

$$L^{P}_{STATEC} = \underbrace{(\underbrace{POP1564 * PART^{s}}_{labour \quad force} * (1 - U^{s}) + FRONT^{s})}_{potential \quad employment \quad (total)} * HOURS^{s} \tag{28}$$

Here, $FRONT$ denotes the cross-border workers, $POP1564$ the population between 15 and 64 years of age, $PART$ the participation rate and U the unemployment rate. The suffix s denotes a detrended (smoothed) variable. Potential national employment is given by the trend labour force multiplied by trend unemployment. Potential employment is the sum of the potential national employment and a trend cross-border workers component. In practise, potential labour is obtained by smoothing participation rates, the number of cross-border workers, hours and the unemployment rate using a HP filter. The main departures from the Commission method are as follows: 1) the (smoothed) number of *frontaliers* workers is added to potential national employment to obtain total potential employment; 2) the long term unemployment rate is computed with a HP filter rather than with the Kalman filter procedure.

The following presents trend TFP computed with the Kalman filter using Luxembourgish national accounts data. The filtered TFP is estimated using the software BGAP (Planas and Rossi, 2009), provided by the Commission on its CIRCA website.

4.1 The potential output with filtered TFP

Recall that 'observed' TFP is computed as a residual, obtained dividing output by the contributions of inputs to production:

$$TFP_t = Y_t / L_t^{\alpha} K_t^{1-\alpha}; \tag{29}$$

Here α is set to 0.52, the sample average of Luxembourg historical data, capital stock (K) is the gross stock and the labour input (L) corresponds to the effective employment discussed in the previous section; t is a time index.

Figure 6 compares the observed growth rates of TFP computed with STATEC data to those obtained by the Commission. The two series are very similar. The series are highly volatile, a well documented feature of Luxembourgish data. Overall, TFP growth has declined over the period. After 1995, negative rates of growth of TFP are recorded in correspondence of the 2001-2003 recession and since the outbreak of the financial crisis. STATEC forecasts a weak growth from 2014 and a more pronounced recovery for 2016, while AMECO's forecasts are negative growth up to 2014.[25]

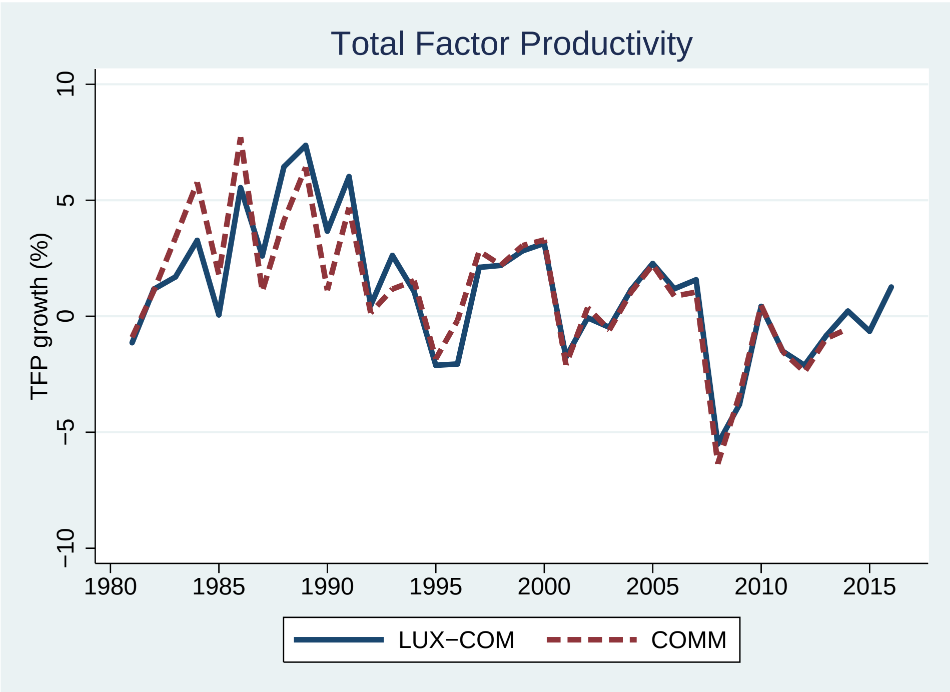

Figure 6: **Total Factor Productivity growth 1980-2016**: STATEC (blue line) and Commission data (red dashed line). (Source: author's computations on STATEC data, Commission.)

Trend TFP is then computed by applying a Kalman filter method on observed values. Potential output is the level of output that corresponds to trend TFP and potential labour input, according to the following equation:

$$Y_p = TFP_p L_p^{\alpha} K^{1-\alpha}; \tag{30}$$

[25] Tendencies discussed here for TFP are also reported in LuxKlems (Peroni, 2012).

(Potential labour input is computed as described in equation 28.) Recall also that the gap measures the difference between observed and potential output, and is computed as follows:

$$Y_{gap} = \frac{Y - Y_p}{Y_p} \tag{31}$$

The gap is negative when observed output is lower than potential output, indicating that the economy does not fully use its productive capacity; viceversa, it is positive when observed output is greater than potential.

Figure 7 presents observed and trend levels of TFP obtained using STATEC's data and the bayesian Kalman filter method adopted by the Commission. One observes the rapid rise in the Luxembourgish TFP which occurred during the 80s. Despite a slow-down in growth rates, the trend remained positive in the subsequent period. One notices the marked decline in TFP levels which occurred since the outbreak of the financial crisis. The forecast years (that is, the year after 2012) mark a very slow recovery. Figure 8 compares the rates of growth of trend TFP obtained by the Commission to those produced by running the Kalman filter on STATEC's data. Trends are similar. The series obtained by STATEC, however, is less smoothed than the Commission's one, and is characterised by a sharper and more pronounced recovery. STATEC's TFP grows at positive rates as from 2016 whereas the Commission's series is negative for the whole projection period (see table 4). This feature may have been produced by different patterns in observed data and the consequent need to adjust the choice of the priors in the bayesian module of the estimation procedure. Section 4.3 discusses in greater detail the impact of the choice of the priors on the results. (Figures 15 and 16 in the appendix B.2 show, respectively, observed and trend TFP growth obtained by the author and by the Commission using the BGAP program.)

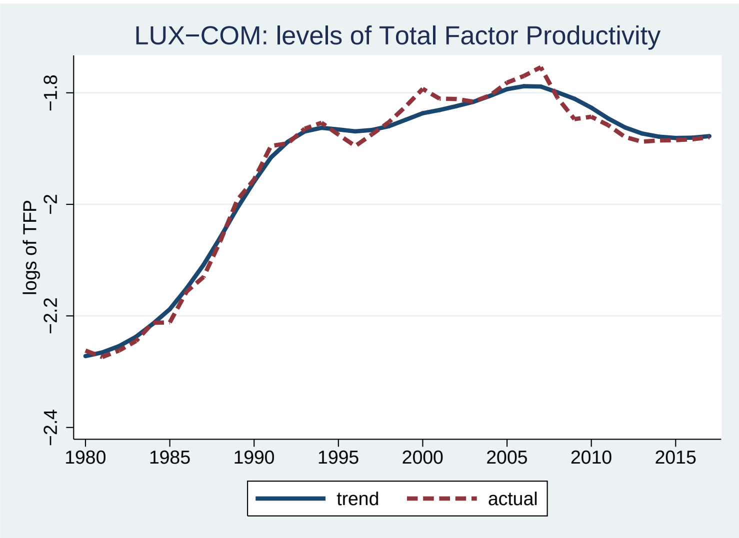

Figure 7: **TFP levels: trend and observed values.** Note: data are in logarithm. (Source: author's computations on STATEC data.)

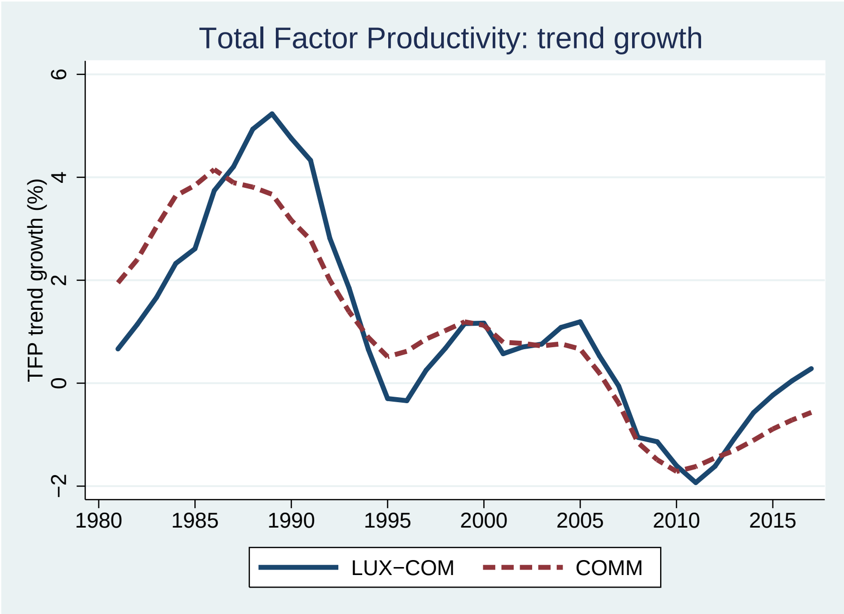

Figure 8: **TFP trend growth:** comparison of STATEC and Commission calculations. Note: the y-axis reports annual percentage growth rates. (Source: STATEC, EU Commission.)

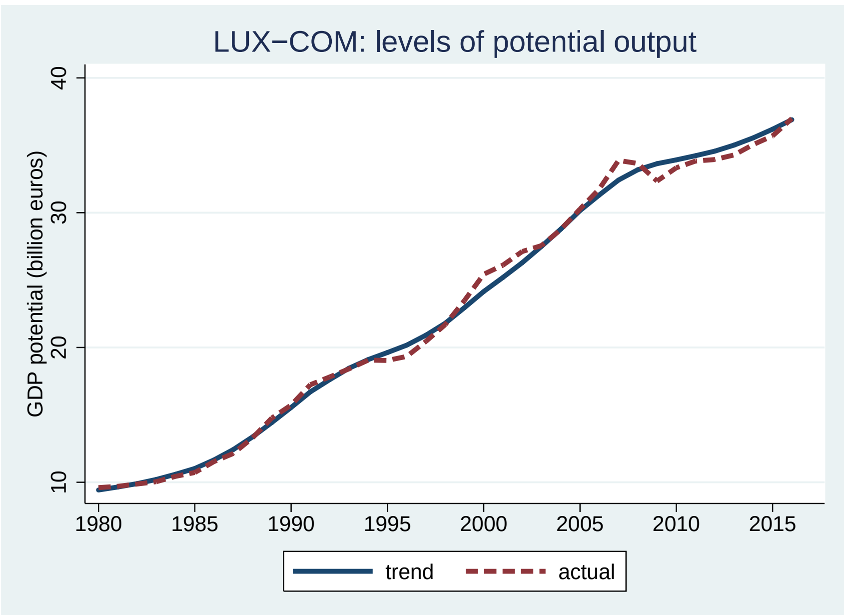

Figure 9: **GDP (levels): observed and potential.** (Source: STATEC.)

Figure 9 presents time series of observed and potential output obtained using the production function methodology on STATEC's data. Figure 10 compares the STATEC's series (continuous blue line) to the one produced by the Commission (red dashed line). Trends are close across datasets, indicating that the method produces robust results. One feature of the data is the variability in potential growth rates. One also observes that overall, since the mid-90s, potential output has grown at rates lower than those recorded in the previous decade, and a further slow-down occurred since the crisis. In the forecasting period, however, STATEC series is characterised by slightly higher potential growth. Despite the Commission being slightly more optimistic during 201 and 2012, STATEC data deliver growth rates of potential output above 1%. In contrast, the Commission predicts growth rates approximately equal to 1%. One can also see that, despite the contraction in potential output that occurred during the crisis, the Kalman filter attributes much of the fall in observed output to cyclical movements in the economy (one can also see figure 18 in the appendix). Finally, figure 11 compares the output gap, computed as a percentage of GDP, obtained at STATEC and by the Commission. The gap estimated with STATEC data is close to the one published by the Commission. It is negative since 2009 (about -4% in that year), indicating that observed output growth is substantially below potential growth since the crisis. In STATEC' series, the gap turns positive in 2016.

Table 7 in the appendix B.4 gives detailed figures for observed output growth, potential output growth and the gap computed with the production function method using STATEC and AMECO data. For comparison, the table reports also series of output growth and the gap computed using a simple HP filter. Figure 17 in appendix B.2 compares rates of growth of potential output obtained with the Kalman filter to those obtained using the HP filter. The evolution is similar across models, although the HP filter delivers a slightly smoother series: the HP filter delivers higher rates of growth at the beginning of the sample, while at the end of the sample gives rates of growth which are closer to the Commission results. The main difference between Kalman filter method and HP filter-based method is that while the first predicts a gradual, albeit slow, recovery in potential growth rates, the latter produce a series characterised by a continuous and more persistent decline.[26]

[26]This feature may be due to the well-known problem of the end-of-period bias of the HP filter, which tends to follow more closely the observed series.

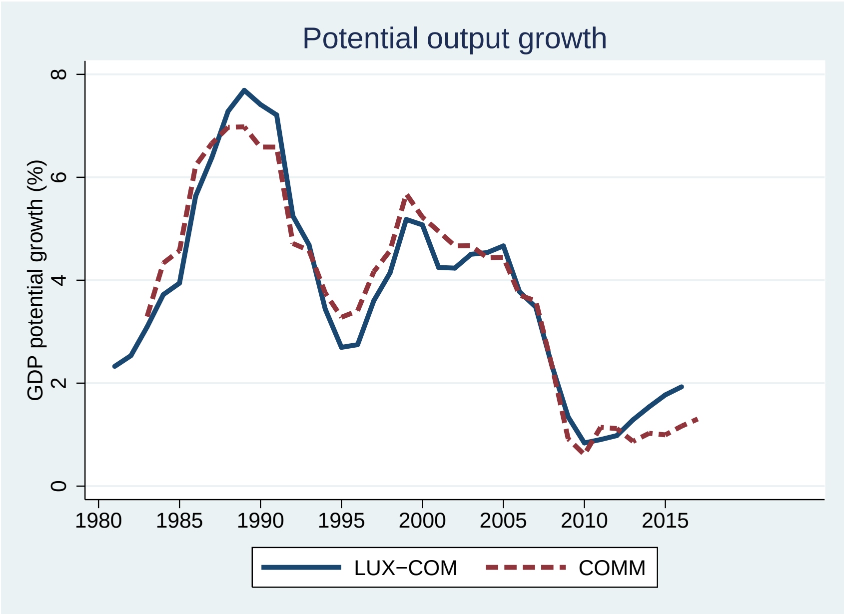

Figure 10: **Potential GDP growth: comparison of STATEC and Commission calculations.** (Source: STATEC, EU Commission.)

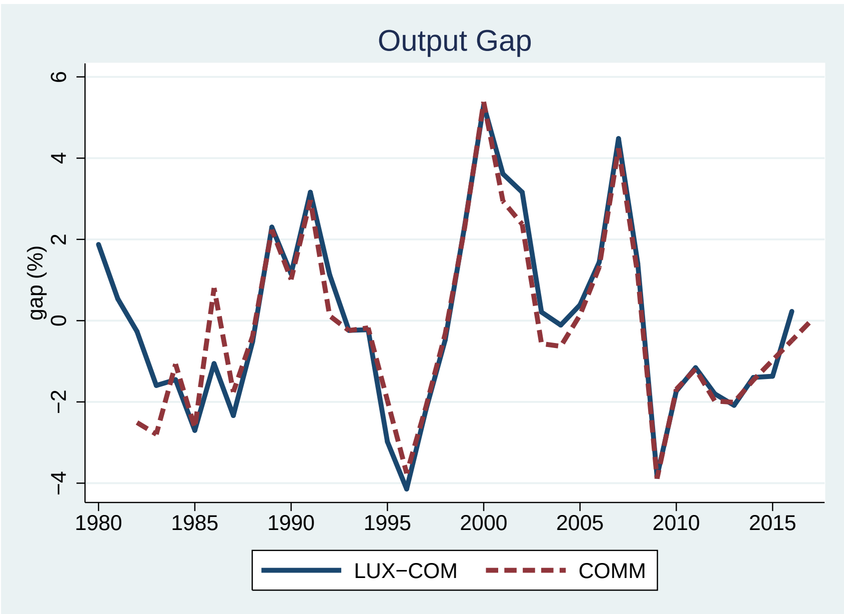

Figure 11: **Output gap (% on potential output): comparison of STATEC and Commission calculations.** (Source: STATEC, EU Commission.)

Table 3: **LUX-COM results**

LU	Gap	GDP growth: actual	GDP growth: potential	Contributions: TFP	Contributions: L	Contributions: K	Labour Potential: POP	Labour Potential: PART	Labour Potential: U
1980	1.88							0.62	2.67
1981	0.54	1.01	2.33	0.67	1.14	0.53	0.79	0.62	2.66
1982	-0.27	1.73	2.53	1.14	1.05	0.34	0.63	0.62	2.64
1983	-1.59	1.75	3.09	1.66	1.17	0.26	0.82	0.62	2.61
1984	-1.45	3.86	3.72	2.33	1.10	0.29	0.60	0.62	2.57
1985	-2.70	2.67	3.94	2.61	1.09	0.24	0.45	0.62	2.52
1986	-1.06	7.31	5.64	3.74	1.33	0.57	0.80	0.62	2.45
1987	-2.34	5.08	6.39	4.21	1.34	0.84	0.73	0.62	2.37
1988	-0.52	9.13	7.28	4.94	1.36	0.98	0.70	0.62	2.31
1989	2.30	10.49	7.69	5.23	1.38	1.07	0.69	0.62	2.26
1990	1.15	6.28	7.41	4.76	1.51	1.14	0.95	0.62	2.24
1991	3.16	9.18	7.21	4.33	1.49	1.39	0.91	0.62	2.25
1992	1.13	3.26	5.25	2.82	1.47	0.96	0.83	0.62	2.28
1993	-0.24	3.32	4.69	1.85	1.49	1.35	0.84	0.63	2.33
1994	-0.22	3.46	3.44	0.66	1.54	1.24	0.83	0.63	2.40
1995	-2.98	-0.10	2.70	-0.30	1.70	1.30	1.02	0.63	2.46
1996	-4.14	1.54	2.75	-0.34	1.74	1.35	0.93	0.63	2.52
1997	-2.19	5.62	3.60	0.25	1.84	1.51	0.99	0.64	2.58
1998	-0.47	5.89	4.14	0.68	1.91	1.56	1.05	0.64	2.67
1999	2.30	7.93	5.18	1.16	2.01	2.01	1.31	0.64	2.77
2000	5.31	7.97	5.08	1.17	2.16	1.75	1.84	0.65	2.91
2001	3.61	2.62	4.25	0.57	1.80	1.88	1.10	0.65	3.09
2002	3.16	3.80	4.24	0.70	1.62	1.91	0.93	0.65	3.30
2003	0.21	1.60	4.50	0.76	1.77	1.98	1.73	0.66	3.55
2004	-0.11	4.22	4.54	1.08	1.52	1.94	1.40	0.66	3.80
2005	0.39	5.16	4.67	1.19	1.58	1.89	1.98	0.66	4.04
2006	1.44	4.82	3.78	0.53	1.38	1.87	1.74	0.66	4.27
2007	4.48	6.43	3.48	-0.06	1.31	2.22	1.93	0.67	4.48
2008	1.41	-0.65	2.34	-1.05	1.31	2.08	2.35	0.67	4.69
2009	-3.83	-3.96	1.34	-1.14	1.08	1.40	2.05	0.67	4.88
2010	-1.73	3.00	0.84	-1.60	0.97	1.47	2.05	0.67	5.06
2011	-1.16	1.49	0.90	-1.93	1.18	1.65	3.00	0.67	5.24
2012	-1.80	0.33	0.98	-1.61	0.84	1.75	2.14	0.67	5.43
2013	-2.08	1.00	1.29	-1.08	0.72	1.64	1.90	0.67	5.60
2014	-1.40	2.24	1.54	-0.57	0.66	1.45	1.78	0.67	5.78
2015	-1.37	1.80	1.77	-0.23	0.63	1.37	1.74	0.67	5.94
2016	0.23	3.54	1.93	0.05	0.60	1.29	1.67	0.67	6.10

Legend: Gap is output gap calculated as percentage of potential output; GDP, both actual and potential, is in annual percent change; POP is population of working age, in annual percent change; PART is trend participation rate, in percent of population of working age; U is trend unemployment rate in percent of labour force. Source: author's computations on STATEC data.

4.2 Growth accounting and robustness

The analysis of the previous section has shown that STATEC's data deliver estimates of potential output higher than those reported by the Commission in the results for the spring forecasting round. To study the sources of this discrepancy we perform a simple growth accounting exercise, that is, we decompose growth in potential GDP in the contributions of potential labour, capital stock, and TFP changes. This is based on the following equation:

$$\Delta ln(Y_p) = \Delta ln(TFP_p) - \alpha \Delta ln(L_p) - (1-\alpha)\Delta ln(K) \tag{32}$$

In each period, the change in the (log of) potential output is the sum of the change in potential TFP and the changes in the potential labour and capital stock parts weighted by the parameters α and $1-\alpha$. Thus, the potential labour contribution on growth is computed by multiplying the wage bill parameter α by the annual change in labour potential; the capital stock part is calculated in an analogous manner:

$$L = \alpha \Delta ln(L_p)$$
$$K = (1-\alpha)\Delta ln(K)$$

Table 4 reports growth rates of potential GDP and its components, namely the TFP, capital and labour potential contributions. One can see that the higher potential growth for the period 2013-2016 is generated by less negative TFP growth than the one reported by the Commission. The dynamics of the capital stock contribution is also more sustained than the one reported by the Commission, while the labour potential contribution is slightly lower than the Commission one. One can also see the sharp increase in trend unemployment which occurred over the period (higher than the one produced by the Commission) and the more optimistic path of population growth.

To check for robustness of results, TFP and potential output have been computed once again leaving the value of the wage bill parameter, α, unchanged with respect to the Commission dataset. (Recall that the Commission set the parameter to 0.65 in place of the value 0.53 found in Luxembourgish data.) Table 8 in the appendix reports the result for potential growth and the gap. One can see that results do not change substantially, despite a slightly higher potential growth generated by the higher value of α; the post-crisis recovery pattern does not differ from the one found in old results.

Table 4: **Growth accounting: potential GDP and its components**

	LUX-COM				COMM			
Year	ΔGDP_{pot}	ΔTFP_{pot}	K	L_{pot}	ΔGDP_{pot}	ΔTFP_{pot}	K	L_{pot}
1990	7.41	4.76	1.14	1.51	6.59	3.17	1.31	1.95
1991	7.21	4.33	1.39	1.49	6.59	2.79	1.74	1.92
1992	5.25	2.82	0.96	1.47	4.71	2.00	0.93	1.71
1993	4.69	1.85	1.35	1.49	4.58	1.39	1.51	1.63
1994	3.44	0.66	1.24	1.54	3.75	0.89	1.34	1.50
1995	2.70	-0.30	1.30	1.70	3.28	0.51	1.17	1.59
1996	2.75	-0.34	1.35	1.74	3.41	0.62	1.19	1.58
1997	3.60	0.25	1.51	1.84	4.17	0.86	1.46	1.82
1998	4.14	0.68	1.56	1.91	4.57	1.02	1.49	2.02
1999	5.18	1.16	2.01	2.01	5.68	1.19	2.10	2.33
2000	5.08	1.17	1.75	2.16	5.23	1.13	1.54	2.50
2001	4.25	0.57	1.88	1.80	4.95	0.80	1.75	2.37
2002	4.24	0.70	1.91	1.62	4.67	0.77	2.03	1.84
2003	4.50	0.76	1.98	1.77	4.67	0.72	2.16	1.77
2004	4.54	1.08	1.94	1.52	4.44	0.76	1.94	1.71
2005	4.67	1.19	1.89	1.58	4.44	0.66	1.90	1.86
2006	3.78	0.53	1.87	1.38	3.71	0.20	1.76	1.75
2007	3.48	-0.06	2.22	1.31	3.60	-0.39	2.25	1.77
2008	2.34	-1.05	2.08	1.31	2.33	-1.16	1.94	1.59
2009	1.34	-1.14	1.40	1.08	0.92	-1.49	1.10	1.33
2010	0.84	-1.60	1.47	0.97	0.61	-1.72	1.20	1.16
2011	0.90	-1.93	1.65	1.18	1.14	-1.62	1.48	1.32
2012	0.98	-1.61	1.75	0.84	1.12	-1.45	1.67	0.94
2013	1.29	-1.08	1.64	0.72	0.87	-1.31	1.48	0.73
2014	1.54	-0.57	1.45	0.66	1.03	-1.11	1.42	0.74
2015	1.77	-0.23	1.37	0.63	1.00	-0.89	1.16	0.74
2016	1.93	0.05	1.29	0.60	1.17	-0.72	1.08	0.82
average	3.48	0.47	1.61	1.40	3.45	0.28	1.56	1.59

Legend: Data are annual % changes in potential GDP and its components. Potential GDP and trend TFP have been obtained using the production function methodology. Note: K and L_{pot} denote, respectively, changes in capital stock contribution and potential labour input contribution. (Sources: author's calculations on STATEC data, EU Commission.)

4.3 Model fitting and sensitiveness to priors

This section looks more closely to the procedure chosen by the Commission to fit the TFP model. Recall that trend and cycle TFP are unobservable variables. The Commission estimates such variables resorting to an unobserved component model, which was discussed in some details in previous sections. The model, which combines time series and economic information to separate short-term fluctutations from long-term movements in the series of interest, is estimated using Bayesian techniques. Broadly speaking, these techniques allow researchers to make explicit assumptions on the behaviour of the parameters/variables of interest, and to "update" such prior knowledge using the information contained in observed data. The prior information is summarised by *prior* probability distribution functions. The goal is to recover the posterior distributions of the variables and parameters of interest.

More formally, assume that the goal of the analysis is to estimate an unknown vector of parameter Θ. Standard statistical analysis proposes a likelihood function method that delivers point estimates for each element of the vector. The point estimates are computed by maximizing the likelihood function $l(X|\Theta)$ where X denotes the data. As said above, Bayesian analysis uses some initial knowledge/guesses on such parameters, $p(\Theta)$, then updates it with information from observed data. (p denotes a probability function.) The combination of these two sources of information gives a posterior distribution which can be factorised as follows:

$$\begin{aligned} p(\Theta|x) &= 1/p(x) \;\dot{}\; p(x|\Theta) \;\dot{}\; p(\Theta) \\ &= constant \;\dot{}\; likelihood \;\dot{}\; prior \end{aligned} \tag{33}$$

The expression above shows that the posterior distribution can be viewed as an augmented likelihood, where the augmentation factor is the prior distribution.[27] It is often not possible to compute the full posterior distribution analytically; if this is the case, numerical techniques are needed. The Commission's framework adopts simulation (sampling) techniques which deliver the modes of the posterior distributions as the Bayesian estimators of the parameters of interest. (That is, the point estimates reported by the Commission are in fact the modes of the posterior distributions.) Note that the simulation strategy adopted by the Commission jointly estimates unobserved variables and the unknown parameters of the model.[28]

The following examines features of the prior distributions of the TFP model's parameters and briefly discusses the fit of the model. We focus on priors and the comparison between priors and posteriors because Bayesian models are usually heavily criticised for their dependence on prior distributions. (The ability of those techniques of updating those distributions with real data, however, is often neglected in the heated debate on the use of Bayesian techniques in econometrics.) Thus, it is important to make us an idea of the role of priors in producing the final results. Table 5 below summarises the distributions, modes, variances and bounds (if applicable) of the priors assigned to the TFP model's parameters for Luxembourg.[29]

Firstly, let us recall the model that fully describes the TFP, as the sum of a cyclical and trend component, plus an observation equation which links the cyclical component of TFP

[27] Equation 34 is a straightforward application of Bayes theorem.

[28] The GAP software implements a Gibbs sampling scheme (Casella and George, 1992). For more details one can refer to Planas et al. (2008); Planas and Rossi (2009).

[29] The priors' choices are discussed in Raciborski (2012). Note that the prior distributions in the table are standard choices in bayesian analysis. Full details on the distributions are available in the appendix of the Commission paper on the production function methodology (D'Auria et al., 2010).

to real data on capacity utilisation:

$$\begin{aligned} u_t &= \mu_U + \beta c_t + e_{Ut}, & var(e) = V_{CU} \\ \Delta p_t &= \mu_{t-1} & \\ \mu_t &= \omega(1-\rho) + \rho\mu_{t-1} + a_{\mu t} & var(a_\mu) = V_\mu \\ c_t &= 2Acos(2\pi/\tau)c_{t-1} - A^2 c_{t-2} + a_{ct} & var(a_c) = V_c \end{aligned}$$

Thus, the parameter vector for the model above is defined as follows:
$\Theta = (\mu_U, \beta, V_{CU}, \omega, \rho, V_\mu, A, \tau, V_c)$. Note that the full vector of unknown variables and parameters is $\Theta' = (c, p; \mu_U, \beta, V_{CU}, \omega, \rho, V_\mu, A, \tau, V_c)$. The following discussion focuses on the parameters listed below:

- τ and A: these give, respectively, the periodicity and amplitude of the cycle;
- ω: this parameter is computed as the historical average rate of growth of countries' TFP. Here, TFP trend growth is set to be 1.5% per year;
- β: gives the strength of the link between capacity utilisation and the TFP cycle. The prior distribution is centered on an elasticity equal to 1.4.

Table 5: **TFP model: prior distribution of parameters for Luxembourg (COMM)**

parameter	dbn.	mean	variance	lower bound	upper bound
τ	B	8	4	2	32
A	B	0.42	0.17		
ω	N	0.015	0.01	0.00	0.03
ρ	N	0.80	0.24	0.00	0.99
β	N	1.40	0.71 $\times V_{CU}$	0.00	5.00
μ_U	N	0.00	0.03 $\times V_{CU}$	-0.10	0.10
V_{CU}	IG	0.004154	0.00415		
V_μ	N	$5.023e(-007)$	$6.4e(-007)$		
V_c	IG	0.001206	0.0008088		

Legend: B denotes the Beta distribution, N the Normal and IG the inverted gamma distributions. (For ρ and β a positive support is imposed.) Source: European Commission https://circabc.europa.eu/.

The first two parameters in the table represent the amplitude, or contraction factor, (A) and periodicity of the cycle (τ). The cycle period is set to 8 years, following Planas and Rossi (2008) and Gerlach and Smets (1999) who found this value to characterise the economic cycles for the Euro-area. This seems adequate to Luxembourg, in the light of previous studies on the country economy's cyclical behaviour and of the degree of openness of the economy (Guarda, 2006; DiMaria, 2008b). The amplitude of the cycle is set to 0.42 (about half of the value chosen for GDP in the cited works).[30]

[30]The upper bound for τ is usually set as the number of available observations. The lowerbound is 2, the miminum periodicity.

The time evolution of the trend TFP is described by the parameters ω and ρ: ω is its unconditional mean, and ρ is the coefficient of autocorrelation. Clearly, ρ is set to be less than 1 to avoid non-stationarity. The setting of the ω prior is discussed at length in Raciborski (2012). This parameter reflects the average historial growth rate of TFP, and is set to 0.015 (that is, a growth rate of 1.5% per year), with an associated standard deviation that allows for a deviation from the mean by 1% per year. The discussion in the OGWG on this prior has focused on two issues: 1) whether to increase the variance of the prior, in order to assign more weight to the data; 2) to link the mean prior more closely to the evolution of TFP, it was also suggested to base its choice on a moving window (for example, select the historical average of the last 15 years of data. In STATEC's series, the average growth rate of TFP is equal to 1.9% for the period 1980-2007 (standard deviation is 2.5), and to 1.2 % for the period 1998-2007 (s.d. equal to 1.5%). The choice of the Commission seems adequate, although the postulated variability of the data is lower than the one actually recorded for Luxembourg. One can also see that the variance of the cycle component (V_c) is of greater magnitude than the variance of the trend component (V_μ) .

The two parameters β and μ represent the slope and the intercept of the equation linking current TFP and the capacity utilisation indicator. The β parameter represents the elasticity of the cyclical component of TFP to the capacity utilisation. A key implication of the model is that this parameter should be greater than one. (An elasticity greater than one signals high responsiveness of capacity use to the cyclical component of TFP, providing support for the empirical specification chosen by the Commission.) β is also restricted to assume positive values.

One problem with Bayesian analysis is the relative scarcity of simple tools to evaluate model fitting compare to standard ML or OLS estimation. (In other words, there are not many ways of telling whether a model provides a good description of the data.) An intuitive way of doing so is to compare the shapes of prior and posterior distributions.[31] Figures 12–13 compare the distributions from the model estimation for β and τ. First of all, the posterior distribution is centered on a value of beta greater than one, providing support to one of the model's main assumptions. The reduction in the posterior's variance is small, which suggests a possible identification problem. A sensitivity analysis, conducted by flattening the prior and modifying its mode, confirms, however, that data are informative. The posterior for cycle periodicity suggests a cycle of slightly larger amplitude than the prior one; the data seem informative, although once again the variance of the posterior distribution does not shrink substantially. The comparison of posterior to prior distribution for the variances of the disturbances signal a good fit. (Graphs are not reported here for reasons of space.) The Geweke's p-values do not signal problems in terms of convergence of the sampling chain.[32]

[31] Posteriors and priors distributions should not differ too much from each other. A failure to do so would point either to model misspecification or to "bad" priors assumptions. Despite this, a shrink in the posterior distribution's variance is desirable compared to the prior; this is because one would expect that the added information derived from the observed data would increase estimation efficiency. At the same time, a posterior distribution that too closely mimics the prior would point to a lack of identification of the parameter. In other words, when prior and posteriors are too similar, the data do not provide relevant information for model fitting and results are completely driven by priors.

[32] Geweke's p-values monitors chain convergence, signalling whether the sampling scheme generates autocorrelation in the simulated data. Another statistics available to evaluate the model performance is the Relative Numerical Efficiency (RNE), which should be close to one (Planas and Rossi, 2009). For the Luxembourgish observation equation, however, the RNE is very small.

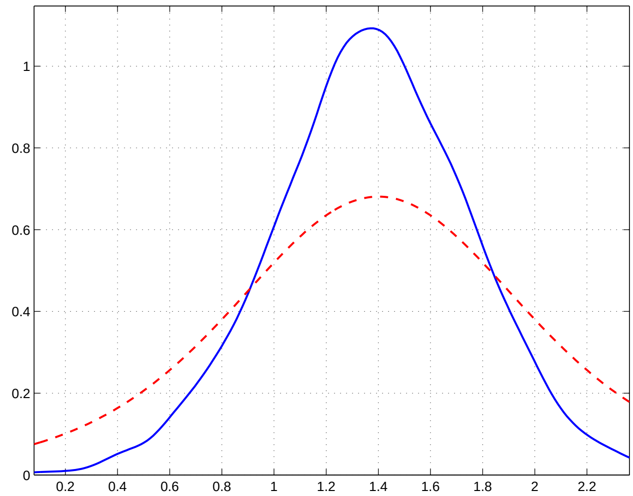

Figure 12: Observation equation: prior (dotted red line) and posterior dbn of Beta parameter (β). (Source: EU Commission.)

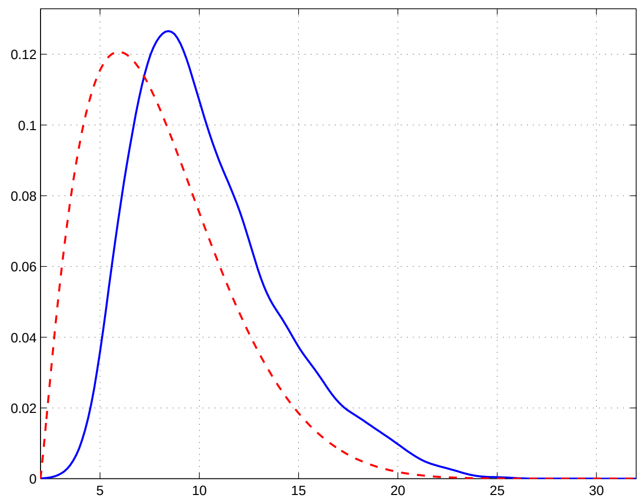

Figure 13: Cycle equation: prior (dotted red line) and posterior dbn of cyclical period (τ). (Source: EU Commission.)

When the Bayesian procedure is run with the data sourced from STATEC, the estimated β is very low and close to zero. Moreover, the posterior distribution for this parameter shows evidence of bimodality (with peaks located close to zero and at 1.5). (This is not reported here for reasons of space.) The parameter τ seems also problematic. The posterior distribution is once again bimodal. One of the peaks of the distribution is located close to the mode of the prior, but the second one corresponds to the much higher value of 17. Increasing the variance of the prior distributions has the effect of reducing the posteriors' variances and attenuate the bimodality in the data. An increase in the variance of the variance parameters (V_{CU}, V_{μ}, V_c) eventually brings results more in line with those published by the Commission and improves the posteriors and the overall fit of the model. (Note that an increase in the variance of the prior distributions amounts to posing less constraints on the posterior distributions, and places more weight on the actual data.) Recall that the final TFP estimates are those reported in figure 8. One observes that STATEC data results in a more volatile trend TFP.

The different outcomes obtained running the Bayesian procedure with different data sources, as well as the different results produced by modifying the priors' variances when using the STATEC dataset, suggests that results should be interpreted with care. One problem is that priors have clearly more weight in small sample, as it is the case here, than in large samples. Indeed, the priors set by the Commission seem too restrictive for the STATEC dataset. The improvement in the posterior distributions obtained by relaxing the priors shows, at least, that the data are informative. Nonetheless, the impact of the prior may be still somewhat overestimated.

Another problem is the possible presence of misspecification of the empirical model chosen by the Commission, particularly in the light of the special characteristics of the Luxembourgish economy. In order to assess the severity of the misspecification problem, more research is needed. This is a widespread concern in the literature on structural modelling, and recently techniques have been devised to assess the relevance of the problem, based on comparing the performance of VARs to structural models. This, however, goes beyond the scope of this article and suggests a possible venue for further research.

5 Conclusions

This article has described the production function approach adopted by the EU Commission to assess output trend and fluctuations for EU member states and discussed its application to the case of Luxembourg. Measures of trend TFP, potential output and the gap obtained by applying the production function methodology to Luxembourgish national account data were presented and compared to the measures published by the Commission in the latest forecasting exercise (Spring 2013).

Firstly, the report has pointed out several differences in the data used by the Commission compared to those available at STATEC:

- The wage share estimated using historical averages for Luxembourg amounts to 0.52, in contrast to the value of 0.65 adopted by the Commission for all member states;
- STATEC dataset includes forecasts up to 2016 while AMECO forecasts variables up to 2014;
- The concept of capital stock adopted at STATEC for computing TFP corresponds to gross capital stock, while the Commission uses AMECO's net stock;
- The participation rates (that is, the ratio of people who are either employed or are actively seeking employment to the total population) are also different. This is due to Statec computing potential labour input by explicitly taking into account the presence of cross-border workers. (The article has pointed out the conceptual difficulties of the concept of potential labour applied to Luxembourg.)

Main results can be summarised as follows. Firstly, results for the projected growth in potential output are more optimistic for STATEC dataset than those produced by the Commission for the years 2013-2016. As a result, the output gap turns positive at the end of the sample (that is, observed output becomes higher than potential). This result is attributed to a less negative dynamics of TFP, which recovers faster than predicted by the Commission. It is also attributed to differences in the variables' forecasts for the period 2013-2016. Secondly, the production function method, based on Kalman filter techniques, produces a different growth path when compared to HP-filtered data. The HP-filter delivers potential output growth rates lower than those produced with the Kalman filter method, and characterised by persistent decline.

The results summarised above, however, should be interpreted with care in view of the high volatility in Luxembourgish data, and, even more, in the light of frequent and substantial data revisions. This makes it difficult to judge the impact of data discrepancies on final results. Thus, one should favour a methodologically sound strategy for evaluating potential output rather than choosing a method on the basis of perceived "more favourable" results based on currently available data. One should also bear in mind the general critique of Orphanides and van Norden (2002) on models that estimate output gaps. These authors also suggested that parameters' instability that characterises unobserved component models aggravates the limitations of the gap estimation.

The article has also highlighted several limitations of the approach chosen by the Commission. The TFP estimates obtained with Bayesian methods are reasonable but seem sensitive to the choice of the priors, a problem that possibly relates to the short time series available. (This latter limitation, however, is shared with many alternative techniques and the Bayesian module may be considered as an attempt to correct for this problem.) Another concern is the possible presence of model misspecification, which is difficult to detect and is left for future research.

In general, the results of this research cast doubts on the one-size-fits-all approach of the Commission. This is true in the light of the special character of the Luxembourgish economy, a small very-open economy and prominent financial center. This also suggests interesting venues for further work. A possible extension/modification of the approach presented in this article is to consider the impact of variables accounting for the financial cycle on Luxembourg, which has been found to improve the measurement of potential output and the gap for other countries (Borio et al., 2013). The NAWRU and labour input model could also be re-considered and developed further to better capture the characteristics of Luxembourg labour market.

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A List of tables & figures

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B Tables & figures

B.1 Net Capital stock

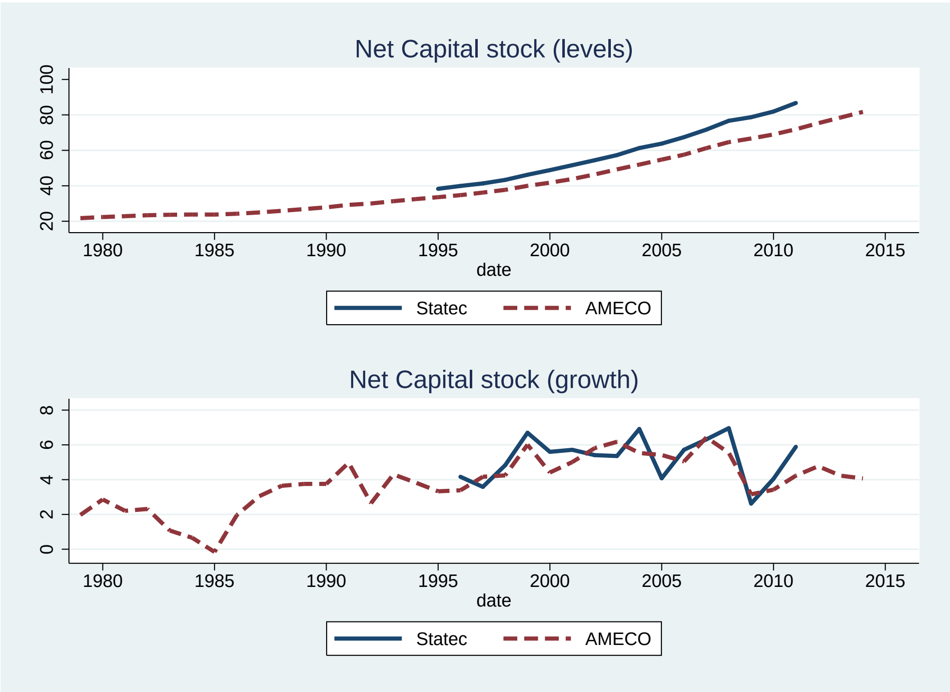

Figure 14: **Net capital stock 1980-2016: levels and (%) growth rates.** STATEC (blue line) and Commission data (dashed red line). (Source: STATEC, AMECO.)

B.2 Trend Total Factor Productivity and Potential output: additional graphs

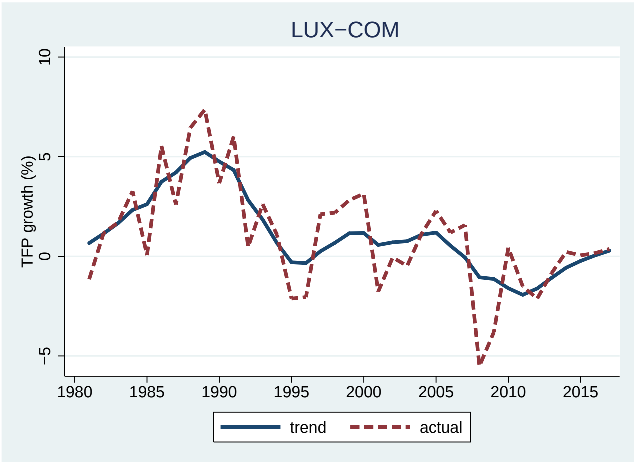

Figure 15: **Trend and observed TFP growth: STATEC data.** (Source: STATEC.)

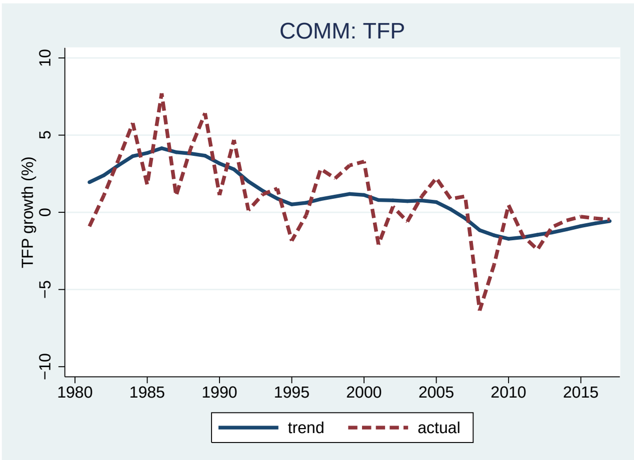

Figure 16: **Trend and observed TFP growth: Commission calculations.** (Source: EU Commission.)

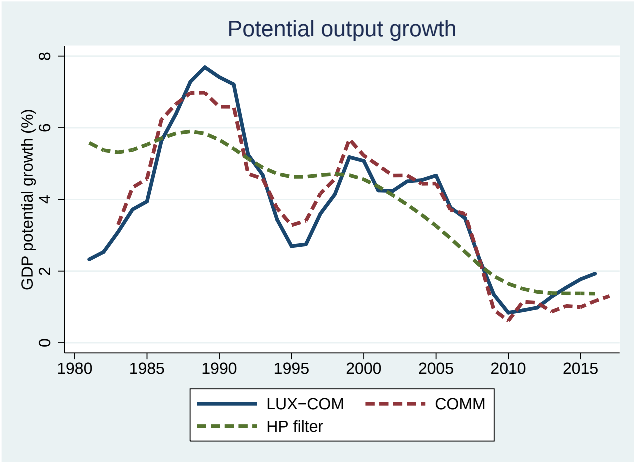

Figure 17: **Potential GDP(levels): comparison of potential GDP growth obtained with Kalman filter (red line, dashed blue line) and HP filter (green line).** (Source: author's computations on STATEC's data, Commission.)

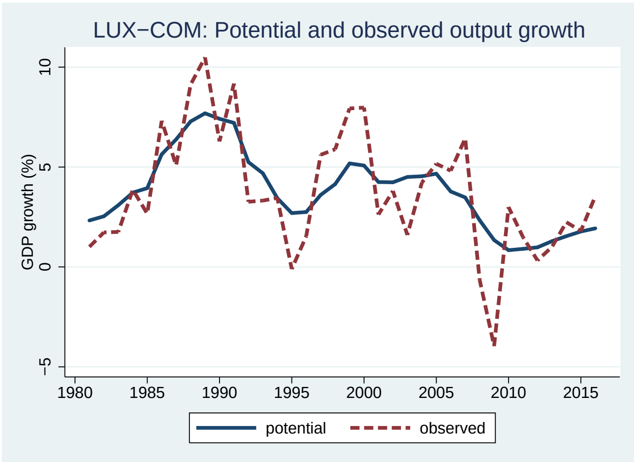

Figure 18: **Potential and observed GDP (growth rates).** (Source: authors computation on STATEC's data.)

B.3 Summary table

Table 6: **Production function: Summary of main methodological assumptions**

	LUX-COM	COMM
Production	Y= $K^{(1-\alpha)} * L^{\alpha}$	Y= $K^{(1-\alpha)} * L^{\alpha}$
α	0.52	0.65
K	gross stock	net stock
L	$(POP * PART^s * (1 - U^s) + FRONT^s) * HOURS^s$	$(FRONT^S + POP * PART^S * (1 - NAWRU)) * HOURS^S$
TFP	$Y/(K^{(1-\alpha)} * L^{\alpha})$	$Y/(K^{(1-\alpha)} * L^{\alpha})$

B.4 Output tables

Table 7: **GDP: observed values, potential and gaps**

Year	LUX-COM			COMM			HP filter	
	ΔGDP	ΔGDP_{pot}	gap	ΔGDP	ΔGDP_{pot}	gap	ΔGDP_{pot}	gap
1980			1.9					9.9
1981	1.0	2.3	0.5	-0.6			5.6	5.0
1982	1.7	2.5	-0.3	1.1		-2.5	5.4	1.2
1983	1.8	3.1	-1.6	2.9	3.3	-2.8	5.3	-2.3
1984	3.9	3.7	-1.5	6.0	4.3	-1.1	5.4	-3.8
1985	2.7	3.9	-2.7	2.9	4.6	-2.6	5.5	-6.5
1986	7.3	5.6	-1.1	9.5	6.2	0.8	5.7	-5.0
1987	5.1	6.4	-2.3	3.9	6.7	-1.8	5.8	-5.7
1988	9.1	7.3	-0.5	8.1	7.0	-0.4	5.9	-2.6
1989	10.5	7.7	2.3	9.3	7.0	2.2	5.8	2.0
1990	6.3	7.4	1.2	5.2	6.6	1.0	5.7	2.7
1991	9.2	7.2	3.2	8.3	6.6	3.0	5.4	6.6
1992	3.3	5.2	1.1	1.8	4.7	0.1	5.1	4.6
1993	3.3	4.7	-0.2	4.1	4.6	-0.2	4.9	3.0
1994	3.5	3.4	-0.2	3.7	3.8	-0.2	4.7	1.7
1995	-0.1	2.7	-3.0	1.4	3.3	-2.0	4.6	-3.0
1996	1.5	2.7	-4.1	1.5	3.4	-3.8	4.6	-6.0
1997	5.6	3.6	-2.2	5.8	4.2	-2.1	4.7	-5.1
1998	5.9	4.1	-0.5	6.3	4.6	-0.3	4.7	-4.0
1999	7.9	5.2	2.3	8.1	5.7	2.3	4.7	-0.8
2000	8.0	5.1	5.3	8.1	5.2	5.4	4.6	2.7
2001	2.6	4.2	3.6	2.5	5.0	2.9	4.4	0.9
2002	3.8	4.2	3.2	4.0	4.7	2.4	4.1	0.6
2003	1.6	4.5	0.2	1.7	4.7	-0.6	3.9	-1.7
2004	4.2	4.5	-0.1	4.3	4.4	-0.6	3.6	-1.0
2005	5.2	4.7	0.4	5.1	4.4	0.1	3.3	0.9
2006	4.8	3.8	1.4	4.8	3.7	1.3	2.9	2.8
2007	6.4	3.5	4.5	6.4	3.6	4.2	2.5	6.9
2008	-0.6	2.3	1.4	-0.7	2.3	1.1	2.2	3.9
2009	-4.0	1.3	-3.8	-4.2	0.9	-3.9	1.9	-2.0
2010	3.0	0.8	-1.7	2.9	0.6	-1.7	1.6	-0.6
2011	1.5	0.9	-1.2	1.6	1.1	-1.2	1.5	-0.6
2012	0.3	1.0	-1.8	0.3	1.1	-2.0	1.4	-1.7
2013	1.0	1.3	-2.1	0.8	0.9	-2.0	1.4	-2.1
2014	2.2	1.5	-1.4	1.6	1.0	-1.5	1.4	-1.3
2015	1.8	1.8	-1.4		1.0	-1.0	1.4	-0.8
2016	3.5	1.9	0.2		1.2	-0.5	1.4	1.3

Legend: ΔGDP_{pot} is annual percentage change in potential GDP, gap is percent on GDP_{pot}; potential TFP obtained with Kalman filter. (Data sources: STATEC, EU Commission.)

Table 8: **Potential GDP: the impact of the α parameter**

Year	GDP actual	LUX-COM ($\alpha = 0.52$)		LUX-COM ($\alpha = 0.65$)		COMM	
Year	Δ	ΔGDP_{pot}	gap	ΔGDP_{pot}	gap	ΔGDP_{pot}	gap
1980			1.88		1.95		
1981	1.01	2.33	0.54	2.49	0.45		
1982	1.73	2.53	-0.27	2.71	-0.53		-2.51
1983	1.75	3.09	-1.59	3.27	-2.03	3.29	-2.80
1984	3.86	3.72	-1.45	3.77	-1.94	4.33	-1.07
1985	2.67	3.94	-2.70	3.94	-3.17	4.58	-2.64
1986	7.31	5.64	-1.06	5.53	-1.43	6.23	0.80
1987	5.08	6.39	-2.34	6.19	-2.51	6.66	-1.75
1988	9.13	7.28	-0.52	7.02	-0.44	6.97	-0.39
1989	10.49	7.69	2.30	7.40	2.68	6.98	2.24
1990	6.28	7.41	1.15	7.23	1.72	6.59	1.02
1991	9.18	7.21	3.16	7.00	3.96	6.59	2.97
1992	3.26	5.25	1.13	5.30	1.86	4.71	0.12
1993	3.32	4.69	-0.24	4.81	0.36	4.58	-0.24
1994	3.46	3.44	-0.22	3.65	0.16	3.75	-0.18
1995	-0.10	2.70	-2.98	2.96	-2.85	3.28	-1.96
1996	1.54	2.75	-4.14	2.91	-4.18	3.41	-3.76
1997	5.62	3.60	-2.19	3.65	-2.27	4.17	-2.12
1998	5.89	4.14	-0.47	4.12	-0.53	4.57	-0.33
1999	7.93	5.18	2.30	4.99	2.44	5.68	2.26
2000	7.97	5.08	5.31	5.00	5.53	5.23	5.38
2001	2.62	4.25	3.61	4.20	3.88	4.95	2.94
2002	3.80	4.24	3.16	4.20	3.46	4.67	2.37
2003	1.60	4.50	0.21	4.60	0.41	4.67	-0.57
2004	4.22	4.54	-0.11	4.55	0.08	4.44	-0.63
2005	5.16	4.67	0.39	4.68	0.56	4.44	0.14
2006	4.82	3.78	1.44	3.76	1.63	3.71	1.32
2007	6.43	3.48	4.48	3.37	4.79	3.60	4.24
2008	-0.65	2.34	1.41	2.36	1.68	2.33	1.12
2009	-3.96	1.34	-3.83	1.57	-3.79	0.92	-3.89
2010	3.00	0.84	-1.73	1.05	-1.89	0.61	-1.69
2011	1.49	0.90	-1.16	1.16	-1.57	1.14	-1.19
2012	0.33	0.98	-1.80	1.13	-2.36	1.12	-1.98
2013	1.00	1.29	-2.08	1.40	-2.74	0.87	-2.01
2014	2.24	1.54	-1.40	1.63	-2.15	1.03	-1.46
2015	1.80	1.77	-1.37	1.85	-2.20	1.00	-0.97
2016	3.54	1.93	0.23	2.00	-0.68	1.17	-0.49

Legend: Results from STATEC data obtained filtering TFP with Kalman method (wage share from the Commission, $\alpha = 0.65$). (Data sources: STATEC, EU Commission.) GDP_{pot} is in level, ΔGDP_{pot} is percentage change in previous year, gap is percentage on GDP_{pot}.

C Estimation of state-space models

This outline is mainly based on Hamilton (1994a).

State-space models are a way of describing the dynamic behaviour of economic variables. A linear state-space representation of a vector Y of dynamic variables is written as follows:

$$\underset{(n \text{ x } 1)}{Y_t} = \underset{(n \text{ x } n)(n \text{ x } 1)}{A \;\; X_t} + \underset{(n \text{ x } r)(r \text{ x } 1)}{B \;\; Z_t} + \underset{(n \text{ x } 1)}{u_t}, \quad u \sim N(0, R) \tag{34}$$

$$Z_{t+1} = FZ_t + v_{t+1} \quad v \sim N(0, Q) \tag{35}$$

The equation 34 is called the **observation equation**. Here, Y is a (n x 1) vector of economic variables. Their dynamic behaviour is described in terms of X, a vector of (possibly) deterministic variables, and Z, the vector (r x 1) of (possibly) unobserved dynamic variables; A and B are two matrices of coefficients; u is an iid vector of measurement errors. (The measurement error vector is sometimes omitted in the literature, in which case relations are identities).[33] The key point here is that the process that determines the dynamic of the variables in Z is known (or assumed known). This dynamic process is described by the **measurement (or state) equation** in 35, usually in terms of a generalised AR(1) process. Finally, u and v are assumed normally distributed in the following, with variance covariance matrix $\Sigma_u = R, \;\; \Sigma_v = Q$.

The following key properties hold:

$$E[Z_{t+k}|Z_t, Z_{t-1}, Z_{t-2}, \dots] = E[Z_{t+k}|Z_t] = F^k Z_t \tag{36}$$

$$E[Y_{t+k}|I_t] = E[AX_{t+k} + BZ_{t+k} + u_{t+k}|I_t] = AX_{t+k} + BE[Z_{t+k}|I_t] = AX_{t+k} + BF^k Z_t \tag{37}$$

Equation 36 tells us that future values of the state vector depends on past values of the state vector only through its current value. In addition, all the relevant information to compute the expected value of Y at time $t + k$ is contained in the information set available in t, $I_t = (Z_t, Z_{t-1}, \dots, X_t, X_{t-1})$, and this is summarised by the value of Z and X in t.

In general, we recall that for a univariate AR(1) process

$$y_{t+m} = \phi^m y_t + \sum_i \phi^{m-i} \epsilon_{t+i} \tag{38}$$

$$E[y_{t+m}|y_t, y_{t-1}, \dots] = E[y_{t+m}|y_t] = \phi^m y_t \tag{39}$$

Clearly this shows: 1) the relevance of this structure for forecasting (Hamilton, 1994a, shows how to write time-series processes using a state-space representation); 2) the dependence of any system on initial values.

Hamilton (1994a) gives two examples of the use of state-space models in applied economics:

- Models with expectations, such as models of real interest rates. In general, models involving rational expectations, with well known time-series properties, naturally lead to state-space representations;

- Economic variables which exhibit cyclical behaviour (Stock and Watson, 1991):

  The dynamic of a vector Y of macroeconomic variables can be explained in terms of an unobserved vector $Z = (c_t, a_{it}, i = 1...n)$, where c denotes the state of the business cycles and a_is are idiosyncratic (random) variables associated to each variable y_i:

[33] X does not need to be deterministic, it suffices to be uncorrelated with u and Z.

$$
\begin{aligned}
Y_t &= \mu + [\Gamma : I_n] \; Z_t && (40)\\
Z_{t+1} &= \Phi Z_t + v_{t+1} && (41)
\end{aligned}
$$

$$
\text{where } [\Gamma : I_n] = \begin{pmatrix} \gamma_1 & 1 & 0 & \dots & 0 \\ \gamma_2 & 0 & 1 & \dots & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ \gamma_n & 0 & 0 & \dots & 1 \end{pmatrix}
$$

C.1 The Kalman filter

In what follows we assume that the coefficients in the matrices A, B, F, Q and R are known.

The main objective here is to compute future values (forecasts) of Z_t given the information set available in $t-1$. The latter is denoted as $I_{t-1} = (x_{t-1}, x_{t-2}, \dots, y_{t-1}, y_{t-2}, \dots)$. The **Kalman filter** is an iterative procedure to generate the series $\hat{Z} = (\hat{Z}_{1|0}, \dots, \hat{Z}_{t|t-1}, \hat{Z}_{t+1|t})$, which exploits properties of conditional expectations and normal distributions. It is important to recall throughout that stationarity is assumed. In what follows the mean of the conditional distribution of Z is denoted by $E[Z_t|I_{t-1}] = \hat{Z}_{t|t-1}$ and its conditional variance by $VAR[Z_t|I_{t-1}] = P_{t|t-1}$. Hamilton describes the steps involved as follows:

- First, it is assumed that the initial value of the state vector Z is drawn from a normal distribution, specified as follows

$$
\begin{aligned}
Z_{1|0} \quad &\sim \quad N(0, P_{1|0}), \text{where} && (42)\\
vec(P_{1|0}) &= [I_{r^2} - (F \otimes F)]^{-1} \; vec(Q)
\end{aligned}
$$

  (The moments of this distributions are unconditional moments, ie, $E(Z_{1|0}) = \hat{Z}_{1|0} = 0.$) In general, because of stationarity, $Z_{t|I_{t-1}} \sim N(\hat{Z}_{t|t-1}, P_{t|t-1})$.

- Then, the conditional distribution of Y is needed:

$$
E(Y_t|X_t, I_{t-1}) = AX_t + B\hat{Z}_{t|t-1} \tag{43}
$$

  The variance is

$$
E\{\underbrace{[Y_t - E(Y_t|X_t, I_{t-1})]}_{forecast \quad error}[Y_t - E(Y_t|X_t, I_{t-1})]^{'}|X_t, I_{t-1}\} = BP_{t|t-1}B^{'} + r \tag{44}
$$

- Thus, the conditional joint distribution of Z_t and Y_t is normal:

$$
\begin{pmatrix} Y_t|X_t, I_{t-1} \\ Z_t|X_t, I_{t-1} \end{pmatrix} \sim N\left(\begin{pmatrix} AX_t + B\hat{Z}_{t|t-1} \\ \hat{Z}_{t|t-1} \end{pmatrix}, \begin{pmatrix} BP_{t|t-1}B^{'} + R & BP_{t|t-1} \\ P_{t|t-1}B^{'} & P_{t|t-1} \end{pmatrix} \right)
$$

- Finally, using the above one obtains the distribution of $Z_t|X_t, Y_t, I_{t-1} = Z_t|I_t \sim N(\hat{Z}_{t|t}, P_{t|t})$, so that we can calculate our final goals, the optimal forecast and its variance:

$$\begin{aligned} \hat{Z}_{t|t-1} = F\hat{Z}_{t|t} = F\hat{Z}_{t|t-1} + \\ +FP_{t|t-1}B(BP_{t|t-1}B^{'} + R)^{-1}(Y_t - AX_t + B\hat{Z}_{t|t-1}) \end{aligned} \quad (45)$$

$$P_{t|t-1} = FP_{t|t}F^{'} + Q = FP_{t|t-1}F^{'} - FP_{t|t-1}B(BP_{t|t-1}B^{'} + R)^{-1}BP_{t|t-1}F + Q \quad (46)$$

If the coefficients of the matrices are not known, then one needs to use a maximum likelihood (or any alternative) procedure to estimate the coefficients first on the basis of an initial guess. This is described in Hamilton (1994a), section 3. Another important tool is the smoothed inference with the Kalman filter, when information from the whole series up to T is used to correct the filtered series in t.

C.2 Summary

The Kalman filter is a computational procedure which updates iteratively a set of initial estimates. It is based on the following elements:

- The set of initial values for observed and unobserved components;
- A correction rule which exploits properties of conditional expectation to construct a filtered series;
- The likelihood function for the measurement equation;

D The software

Information on software and data is publicly available at

`http://circa.europa.eu/Public/irc/ecfin/outgaps/library`

The methodology described in this report is implemented by means of the following software:

1. The software BGAP and GAP (Planas and Rossi, 2009), which estimate the filtered Solow Residual and the NAWRU, are downloadable at

   `http://eemc.jrc.ec.europa.eu/Software-GAP.htm`

2. A set of RATS(Estima) codes for estimating the potential growth rates and output gaps.