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Abstract

Confronted with negative environmental impacts, rising fuel costs and increasing congestion, many cities are implementing sustainable mobility measures to improve the flow of passenger and goods. Examples of these measures are use of public transport, cycling, walking, energy efficient vehicles, biofuels. The challenge before transport decision makers is which one(s) to choose for implementation as often there is no or limited quantitative data available on the subject. Moreover, the context of each city, its geographic and transport conditions restrict the generalization of results obtained in experienced cities. In this paper, we investigate four multicriteria decision making (MCDM) techniques namely TOPSIS, VIKOR, SAW and GRA for sustainability evaluation of urban mobility projects under qualitative data and demonstrate their application through a numerical example.

Keywords: Multicriteria decision making, Urban Mobility, Sustainability Evaluation, TOPSIS, VIKOR, GRA, SAW, Fuzzy Numbers.

JEL classification codes: C6, D8, R4

1. Introduction

Sustainable transportation is vital for modern cities to ensure mobility of goods, people and a healthier environment. More and more cities are becoming active in this direction and implementing measures that encourage the use of public transport and softer modes of transport such as walking, biking etc. Luxembourg city transportation group has also implemented several such measures to improve mobility and modal split towards public transport. Modal split represents the distribution (in percentage) of travellers with respect to usage of different modes of transport (e.g. bus, tram, private car, cycling). Modal split in favour of public transport will improve city sustainability. The Luxembourg authority (Ministry of Sustainable Development and Infrastructure) is aiming to achieve a modal split of 75/25 in 2020 (75% of trips by private vehicles and 25% by public transportation). In 2007, the modal split was 85.5/14.5 (source: MODU strategy). To achieve this target, the Luxembourg Government is planning several transport projects. Among them is the implementation of a tramway in Luxembourg City in 2017. The tramway project will involve implementation of several measures like the reorganization of bus lines and train networks to achieve a better correspondence of transport supply and demand. These transport projects will affect the mobility of the trans-border commuters in particular (Omrani et al., 2010) and therefore, it is important to perform careful evaluation and selection of these projects for sustainable mobility planning.

In this paper, we are investigating the use of multicriteria decision making techniques for sustainability evaluation of urban mobility projects. The Bruntland Commission defines sustainability as development that meets the needs of the present without compromising the ability of future generations to meet their own needs (United Nations World Commission on Environment & Development, 1987). The World Bank (1996) uses the “triple bottom line” of economic, environmental, and social equity to define sustainability. In this paper, we integrate the triple bottom line concept i.e., achievement of social, economical and environmental objectives with technical efficiency for evaluating urban mobility projects through the use of multicriteria decision making (MCDM) techniques.

Multicriteria decision making involves evaluating a set of candidates (alternatives) against a set of given criteria by a committee of decision makers. Multicriteria decision making has been widely applied in evaluating transportation projects (Stewart 1994, Teng and Tzeng 1996, Won 1990, Shang et al., 2004, Omrani et al., 2006, Zak 2011). Yedla and Shrestha (2009) use AHP to evaluate six sustainable transportation modes. Tsamboulas and Mikroudis (2000) present a multi-criteria evaluation framework of environmental impacts and costs of transport initiatives. Awasthi and Omrani (2009) present an AHP and belief theory based approach for evaluating sustainable transportation solutions. Avineri et al (2000) perform transportation project selection using fuzzy sets theory. Eboli and Mazzulla propose a multicriteria methodology for evaluating transit service quality based on subjective and objective measures from the passenger's point of view. Friesz and Tourreiles (1981) perform comparison of multicriteria optimization methods in transport project evaluation. Frohwein et al (1999) propose a multicriteria framework to aid comparison of roadway improvement projects. Giuliano (1985) propose a multi-criteria method for transportation investment planning. Most of these studies rely on a single MCDM method without validating the results with other MCDM techniques. Under this situation, it is hard to see the impact of MCDM technique selection on the final results. This is the challenge we are addressing in this paper.

The rest of the paper is organized as follows. In section 2, we present the problem definition. The solution approach is provided in section 3. Section 4 presents the numerical application of the proposed approach. Finally, in section 5 we provide the conclusions and steps for future work.

2. Problem Definition

Our goal in this paper is to investigate the application of multicriteria decision making for sustainability evaluation of urban mobility projects. To achieve this goal, the research issues we are addressing are:

- Which evaluation criteria to choose for sustainability evaluation ? How to obtain them ?
- How to generate project ratings under lack of quantitative data ?

- Which multicriteria method to choose for project evaluation ? How to deal with qualitative ratings in the selected method?

3. Solution Approach

Our solution approach comprises of three main steps.

1. Selection of evaluation criteria using literature review
2. Generating qualitative criteria and alternative ratings using expert opinions
3. Identifying the best alternative using various multicriteria decision making techniques

3.1 Selection of evaluation criteria

The first step involves selection of criteria for evaluating sustainability of urban mobility projects through comprehensive literature review (Jeon, Amekudzi and Guensler 2008, Jonsson 2008, Litman 2009, Levine and Underwood, 1996, Meyer and Miller, 2001, Nakanishi, 1997, Nathanail 2008, Nocera, 2010, Richardson, 2005, Zietsman 2000, Sayers et al., 2003), and our practical experience with city transportation projects (ECOSYMPA and SUCCESS) in La Rochelle, France. The final list contains 19 criteria (Table 1).

Category	***Criteria***	***Type***
Economic	1. Revenues	B
	2. Operating costs	C
Environmental	3. Energy conservation	B
	4. Conformance to environmental standards	B
Technical	5. Possibility of expansion	B
	6. Service Network	B
	7. Occupancy rate	B
	8. Number of users	B
	9. Mobility	B
	10. Travel cost	C
	11. Service Reliability	B
	12. Travel time	B
	13. Accessibility	B
	14. Customer Responsiveness	B
	15. Connectivity to multimodal Transport	B
	16. Adapted to customers with Specific Needs	B
Social	17. Gender Equity	B
	18. Labor Welfare	B
	19. Ethics/Fairtrade Practice	B

Table 1 : Sustainability Evaluation Criteria

It can be seen in Table 1, that criterion C2 and C10 are the cost (C) category criteria that is, the lower the value, the more sustainable the alternative (or urban mobility project). The remaining criteria are benefit (B) type criteria, that is, the higher the value, the more sustainable the urban mobility project.

3.2 Generating qualitative criteria and alternative ratings.

For sustainability evaluation of urban transportation projects, we need data on social-economic-environmental-technical characteristics (or criteria). However, it has been observed in general practice, that often there is almost none or very limited data available on this subject, thereby making the evaluation process difficult. To address this situation, we will make use of qualitative ratings such as Good, Very Good, Poor, Very Poor etc. for assessing the alternatives and the criteria. Later, they will be transformed into fuzzy numbers using conversion schemes provided in Table 2 and Table 3 for further processing through MCDM techniques.

3.2.1 Preliminaries of fuzzy set theory

Definition 1: A triangular fuzzy number is represented as a triplet $\tilde{a} = (a_1,a_2,a_3)$. Due to their conceptual and computation simplicity, triangular fuzzy numbers are very commonly used in practical applications (Pedrycz 1994, Klir and Yuan 1995). The membership function $\mu_{\tilde{a}}(x)$ of triangular fuzzy number $\tilde{a}$ is given by:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, x \le a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, a_1 < x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, a_2 < x \le a_3, \\ 0, x > a_3 \end{cases}$$

Where a_1, a_2, a_3 are real numbers and $a_1<a_2<a_3$. The value of x at a_2 gives the maximal grade of $\mu_{\tilde{a}}(x)$ i.e., $\mu_{\tilde{a}}(x) = 1$; it is the most probable value of the evaluation data. The value of x at a_1 gives the minimal grade of $\mu_{\tilde{a}}(x)$ i.e., $\mu_{\tilde{a}}(x) = 0$; it is the least probable value of the evaluation data. The narrower the interval $[a_1,a_3]$, the lower is the fuzziness of the evaluation data.

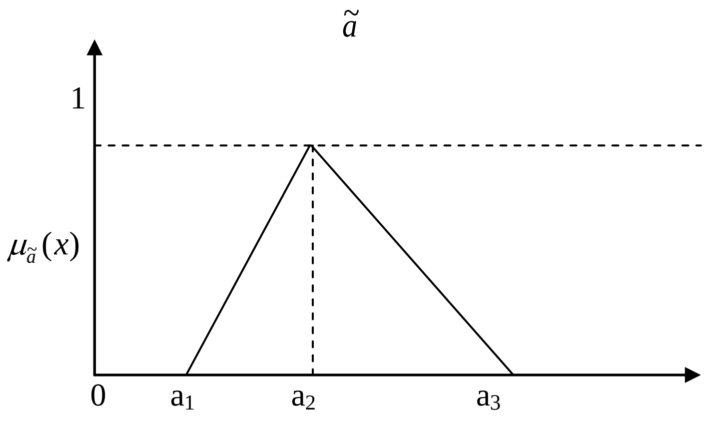

Figure 1: Triangular fuzzy number $\tilde{a}$

Definition 2: In fuzzy set theory, conversion scales are applied to transform the qualitative terms into fuzzy numbers. Table 2 and 3 present the conversion schemes for the qualitative alternative and criteria ratings.

Qualitative Rating	***Membership Function***
Very poor (VP)	(1,1,3)
Poor(P)	(1,3,5)
Fair (F)	(3,5,7)
Good(G)	(5,7,9)
Very Good (VG)	(7,9,9)

Table 2: Fuzzy transformation for qualitative alternative ratings

Qualitative Rating	***Membership Function***
Very Low	(1,1,3)
Low	(1,3,5)
Medium	(3,5,7)
High	(5,7,9)
Very High	(7,9,9)

Table 3: Fuzzy transformation for qualitative criteria ratings

3.3 Multicriteria decision making

The third step involves multicriteria decision making for sustainability evaluation of urban mobility projects. We have chosen four techniques namely Fuzzy TOP-SIS, Fuzzy VIKOR, Fuzzy SAW and Fuzzy GRA in this paper. These techniques were particularly chosen because they perform alternative evaluation based on the closeness to ideal solution (except Fuzzy SAW).

Let us consider a set of *m* alternatives called $A = \{A_1, A_2, ., A_m\}$ which are to be evaluated against a set of *n* criteria, $C = \{C_1, C_2, ., C_n\}$. The criteria weights are denoted by $w_j (j=1,2,..,n)$. The performance ratings of decision maker $D_k (k = 1,2,.., K)$ for each alternative $A_i (i=1,2,..,m)$ with respect to criteria $C_j (j = 1,2,.., n)$ are denoted by:

$\tilde{R}_k = \tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk}), i = 1,..,m; j = 1,2,..,n; k = 1,2,..K$ with membership function $\mu_{\tilde{R}_k}(x)$.

If the fuzzy ratings of k decision makers are described by triangular fuzzy number $\tilde{R}_k = (a_k, b_k, c_k), k = 1,2,.., K,$ then the aggregated fuzzy rating is given by $\tilde{R} = (a,b,c), k = 1,2,.., K$ where;

$$a = \min_k\{a_k\}, b = \frac{1}{K}\sum_{k=1}^{K} b_k, c = \max_k\{c_k\} \tag{1}$$

If the fuzzy rating of the k^{th} decision maker for alternative A_i and criteria C_j are given by $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ and the importance weight by $\tilde{w}_{jk} = (a_{jk}, b_{jk}, c_{jk}), i = 1,2,,, m, j = 1,2,..n$ respectively, then the aggregated fuzzy ratings ($\tilde{x}_{ij}$) of alternatives with respect to each criteria based on eqn (1) are given by $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ where

$$a_{ij} = \min_k\{a_{ijk}\}, b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk}, c_{ij} = \max_k\{c_{ijk}\} \tag{2}$$

The aggregated fuzzy weights ($\tilde{w}_j$) of each criterion are calculated as $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ where

$$w_{j1} = \min_k\{w_{jk1}\}, w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2}, w_{j3} = \max_k\{c_{jk3}\} \tag{3}$$

The fuzzy decision matrix for the alternatives ($\tilde{D}$) and the criteria ($\tilde{W}$) is constructed as follows:

$$\tilde{D} = \begin{matrix} & C_1 & C_2 & & C_n \\ A_1 & \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ A_2 & \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{matrix} \quad ,i=1,2,\dots,m;\ j=1,2,..,n \tag{4}$$

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, .., \tilde{w}_n) \tag{5}$$

The goal is now to generate alternative ratings based on $\tilde{D}$ and $\tilde{W}$ and rank them. The stepwise description for each of the four MCDM techniques for generating these ratings is presented as follows.

3.3.1. Fuzzy VIKOR

The fuzzy VIKOR technique involves fuzzy assessments of criteria and alternatives in VIKOR (in Serbian: VlseKriterijumska Optimizacija I Kompromisno Resenje). Its foundation lies in finding a compromise solution [Opricovic, 1998]. It measures the closeness of the alternative with respect to the positive ideal solution for evaluation.

Step 1: Defuzzify the elements of fuzzy decision matrix for the criteria weights and the alternatives into crisp values. A fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can be transformed into a crisp number a by employing the below equation:

$$a = \frac{a_1 + 4a_2 + a_3}{6} \tag{6}$$

Step 2: Determine the best f_j^* and the worst values f_j^- of all criteria ratings j=1,2,...,n

$$f_j^* = \max_i \{x_{ij}\} \tag{7}$$

$$f_j^- = \min_i \{x_{ij}\}$$

Step 3: Compute the values S_i and R_i using the following equations

$$S_i = \sum_{j=1}^{n} w_j \frac{f_j^* - x_{ij}}{f_j^* - f_j^-} \tag{8}$$

$$R_i = \max_j w_j \frac{f_j^* - x_{ij}}{f_j^* - f_j^-}$$

Step 4: Compute the values Q_i as following

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1-v) \frac{R_i - R^*}{R^- - R^*} \tag{9}$$

where:

$S^* = \min_i S_i;$
$S^- = \max_i S_i;$
$R^* = \min_i R_i;$
$R^- = \max_i R_i;$ (10)

And v is the weight for the strategy of maximum group utility and 1-v is the weight of the individual regret.

Step 5: Rank the alternatives, sorting by the values *S*,*R* and *Q* in ascending order.

Step 6: Propose as a compromise solution the alternative ($A^{(1)}$) which is the best ranked by the measure *Q*(minimum) if the following two conditions are satisfied

C1: Acceptable advantage

$$Q(A^{(2)}) - Q(A^{(1)}) \geq DQ \quad (11)$$

Where $A^{(2)}$ is the alternative with second position in the ranking list by *Q* and

$DQ = 1/J\text{-}1$ (12)

C2: Acceptable stability in decision making

The alternative $A^{(1)}$ must also be the best ranked by *S* or/and *R*. The compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when v>0.5 is needed), or "by consensus $v \approx 0.5$", or "with veto" (v<0.5). Please note that v is the weight of the decision making strategy of maximum group utility.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied Or
- Alternatives $A^{(1)}, A^{(2)}, ..., A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the position of these alternatives are in closeness).

3.3.2 Fuzzy TOPSIS

The fuzzy TOPSIS approach involves fuzzy assessments of criteria and alternatives in TOPSIS [Hwang and Yoon, 1981]. The TOPSIS technique chooses an alternative that is closest to the positive ideal solution and farthest from the negative ideal solution. A positive ideal solution is composed of the best performance values for each criterion whereas the negative ideal solution consists of the worst performance values.

Step 1: Normalize the fuzzy decision matrix

The raw data are normalized using linear scale transformation to bring the various criteria scales into a comparable scale. The normalized fuzzy decision matrix $\tilde{R}$ is given by:

$$\tilde{R} = [\tilde{r}_{ij}]_{mxn}, i = 1,2,..,m; j = 1,2,..,n \quad (13)$$

where:

$$\tilde{r}_{ij} = (\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}) \text{ and } c_j^* = \max_i c_{ij} \qquad \text{(benefit criteria)} \quad (14)$$

$$\tilde{r}_{ij} = (\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}) \text{ and } a_j^- = \min_i a_{ij} \qquad \text{(cost criteria)} \quad (15)$$

Step 2: Compute the weighted normalized matrix

The weighted normalized matrix $\tilde{V}$ for criteria is computed by multiplying the weights ($\tilde{w}_j$) of evaluation criteria with the normalized fuzzy decision matrix $\tilde{r}_{ij}$.

$$\tilde{V} = [\tilde{v}_{ij}]_{mxn}, i = 1,2,..,m; j = 1,2,..,n \text{ where } \tilde{v}_{ij} = \tilde{r}_{ij}(.)\tilde{w}_j \quad (16)$$

Step 3: Compute the fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS) for each criteria:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, .., \tilde{v}_n^*) \text{ where } \tilde{v}_j^* = \max_i \{v_{ij3}\}, i = 1,2.., m; j = 1,2,.., n \quad (17)$$

$$A^- = (\tilde{v}_1, \tilde{v}_2, .., \tilde{v}_n) \text{ where } \tilde{v}_j^- = \min_i \{v_{ij1}\}, i = 1,2.., m; j = 1,2,.., n \quad (18)$$

Step 4: Compute the distance of each alternative from FPIS and FNIS:

The distance (d_i^*, d_i^-) of each weighted alternative $i = 1,2..,m$ from the FPIS and the FNIS is computed as follows:

$$d_i^* = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^*), i = 1,2,..,m \quad (19)$$

$$d_i^- = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1,2,..,m \quad (20)$$

Where $d_v(\tilde{a}, \tilde{b})$ is the distance measurement between two fuzzy numbers $\tilde{a}$ and $\tilde{b}$ and $d_v(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]}$ (21)

Step 5: Compute the closeness coefficient (CC_i) of each alternative.

The closeness coefficient CC_i represents the distances to the fuzzy positive ideal solution (A^*) and the fuzzy negative ideal solution (A^-) simultaneously. The closeness coefficient of each alternative is calculated as:

$$CC_i = \frac{d_i^-}{d_i^- + d_i^*}, i = 1,2,..,m \quad (22)$$

Step 6: Rank the alternatives

In step 9, the different alternatives are ranked according to the closeness coefficient (CC_i) in decreasing order. The best alternative is closest to the FPIS and farthest from the FNIS.

Note: It can be seen from the above equations that VIKOR method uses linear normalization whereas TOPSIS method uses vector normalization. Also, the aggregation function used by VIKOR represents the distance from the ideal solution whereas TOPSIS method uses distances from the ideal and anti-ideal solutions. The relative importance of these distances is not considered in TOPSIS.

3.3.3 Fuzzy Simple Aggregated Weighting

The fuzzy SAW technique involves fuzzy assessments of criteria and alternatives in SAW (Simple Additive Weighting) and uses an overall score of the alternative using weighted sum of its attribute values for alternative selection (Yoon and Hwang, 1981).

Step 1: Normalize the data using the following equations.

$$r_{ij} = \frac{x_{ij}}{\max_i \{x_{ij}\}} \forall i = 1,2,..,m;\ j = 1,2,..,n \text{ (benefit type criteria)} \tag{23}$$

$$r_{ij} = \frac{\min_i \{x_{ij}\}}{x_{ij}} \forall i = 1,2,..,m;\ j = 1,2,..,n \text{ (cost type criteria)} \tag{24}$$

Step 2: Calculate the overall performance rating u_i for alternative *i* by aggregating the product of its various criteria values with their respective weights.

$$u_i = \sum_j w_j x_{ij} \tag{25}$$

Step 3: Select the alternative with the highest overall performance value $u_i \forall i = 1,2,..m$

3.3.4. Fuzzy Grey Relational Analysis

The fuzzy GRA technique involves fuzzy assessments of criteria and alternatives in GRA (grey Relational Analysis) and uses the correlation between the alternative and the ideal alternative (reference sequence) to generate alternative rankings. The closer the alternative is to the ideal alternative, the better it is.

Step 1: Normalize the data using the following equations:

$$\tilde{r}_{ij} = \frac{x_{ij} - Min\{x_{ij}, i = 1,2,..,m\}}{Max\{x_{ij}, i = 1,2,..,m\} - Min\{x_{ij}, i = 1,2,..,m\}}, \forall j = 1,2,.,n \text{ (Benefit type criteria)} \tag{26}$$

$$= \frac{Max\{x_{ij}, i = 1,2,..,m\} - x_{ij}}{Max\{x_{ij}, i = 1,2,..,m\} - Min\{x_{ij}, i = 1,2,..,m\}}, \forall j = 1,2,.,n \text{ (Cost type criteria)} \tag{27}$$

$$= \frac{| x_{ij} - x_j^* |}{Max\{Max\{x_{ij}, i = 1,2,..,m\} - x_j^*, x_j^* - Min\{x_{ij}, i = 1,2,..,m\}\}}, \forall i, = 1,2,..,m;\ j = 1,2,.,n$$

(The closer to the goal type criteria) (28)

Step 2: Calculate the reference sequence for the normalized criteria. The reference sequence $X_0 = (x_{01}, x_{02}, x_{03}, ... x_{0j},, x_{0n})$ contains the ideal value for each criterion. For cost category criteria, it is the lowest value whereas for the benefit category criteria, it is the highest value. Our aim is to find the alternative whose comparability sequence is closest to the reference sequence.

Step 3: Calculate the grey relational coefficient $\gamma(x_{0j}, x_{ij})$ between x_{ij} and x_{0j} to determine the closeness of x_{ij} to x_{0j}.The larger the grey relational coefficient, the closer x_{ij} and x_{0j} are. The grey relational coefficient is calculated as follows:

$$\begin{aligned}
&\gamma(x_{0j}, x_{ij}) = \frac{\Delta_{\min} + \zeta\Delta_{\max}}{\Delta_{ij} + \zeta\Delta_{\max}} \forall i = 1,2,...,m;\ j = 1,2..,n \\
&\Delta_{ij} = \left| x_{0j} - x_{ij} \right| \\
&\Delta_{\min} = Min\{\Delta_{ij}, i = 1,2,...,m;\ j = 1,2..,n\}, \\
&\Delta_{\max} = Max\{\Delta_{ij}, i = 1,2,...,m;\ j = 1,2..,n\},
\end{aligned} \qquad (29)$$

where ζ is the distinguishing coefficient, $\zeta \in [0,1]$. The purpose of distinguishing coefficient is to expand or compress the range of grey relational coefficient.

Step 4: Calculate the grey relational grade $\Gamma(X_0, X_i)$ between X_0 and X_i. The grey relational grade represents the level of correlation between the reference sequence and the comparability sequence and is given by.

$$\Gamma(X_0, X_i) = \sum_{j=1}^{n} w_j \gamma(x_{0j}, x_{ij}), i = 1,2,..,m \qquad (30)$$

Where w_j is the weight of the attribute *j* and $\sum_{j=1}^{n} w_j = 1$.

Step 5: Select the alternative with the highest grey relational grade.

The ranking generated by the four methods will be subject to veto rule. The alternative (s) that is ranked highest by the majority of techniques will be finally chosen.

4. NUMERICAL APPLICATION

In this section, we present the application of four MCDM techniques namely Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy GRA, Fuzzy SAW for sustainability evaluation of three urban mobility projects (A1, A2, A3) in the context of city of Luxembourg. Examples of these projects are implementation of new tramway in the city center (A1), reorganization of existing bus lines in the city to perform optimized service (A2), and implementation of electric vehicle car-sharing stations in the city (A3).

A committee of three decision makers (D1, D2, and D3) is formed to obtain qualitative ratings (Tables 2 and 3) for the criteria and the alternatives. These ratings are presented in Table 4.

Criteria	**Qualitative rating**			***Aggregate Fuzzy Rating***	***Crisp rating***
	D1	**D2**	**D3**		
C1	L	VL	M	(1,3,7)	3.333
C2	H	L	M	(1,5,9)	5
C3	VL	VL	VH	(1,3.667,9)	4.111
C4	H	L	L	(1,4.333,9)	4.556
C5	VH	L	L	(1,5,9)	5
C6	VL	L	H	(1,3.667,9)	4.111
C7	L	H	M	(1,5,9)	5
C8	L	L	VL	(1,2.333,5)	2.556
C9	VL	M	L	(1,3,7)	3.333
C10	M	L	L	(1,3.667,7)	3.778
C11	M	VL	H	(1,4.333,9)	4.556
C12	M	L	H	(1,5,9)	5
C13	M	L	VL	(1,3,7)	3.333
C14	VH	VH	VL	(1,6.333,9)	5.889
C15	H	VH	H	(5,7.667,9)	7.444
C16	L	L	L	(1,3,5)	3
C17	M	VL	H	(1,4.333,9)	4.556
C18	H	M	VL	(1,4.333,9)	4.556
C19	H	M	M	(3,5.667,9)	5.778

Table 4: Qualitative Assessments and Aggregate fuzzy criteria ratings

The aggregated fuzzy weights (w_{ij}) of criteria are obtained using Eq. (3). For example, for criteria C1 (Qualitative Rating = (L,VL,M)), the aggregated fuzzy weight is given by $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ where:

$$w_{j1} = \min_k(1,1,3), w_{j2} = \frac{1}{3}(3+1+5), w_{j3} = \max_k(5,3,7)$$
$$\tilde{w}_j = (1,3,7)$$

The aggregated fuzzy weights $\tilde{w}_j$ are transformed into crisp number w_j using eq (6). For example, for criteria C1, $\tilde{w}_j = (1,3,7)$, we have $w_j = \frac{1*1+(4*3)+7}{6} = 3.33$. Likewise, we compute the aggregate weights for the remaining criteria. The results for aggregate weights of the 19 criteria are presented in last column of Table 4. Table 5 presents the qualitative ratings for the three alternatives provided by the decision making committee.

	A1			**A2**			**A3**		
Criteria	D1	D2	D3	D1	D2	D3	D1	D2	D3
C1	L	L	VH	VL	VH	VL	H	H	L
C2	VL	L	VL	VL	VL	M	H	L	VL
C3	H	VH	VL	VH	VH	M	VH	H	VL
C4	L	VL	VH	VL	VL	M	H	H	VL
C5	M	H	L	H	M	M	H	VL	VL
C6	VH	L	VH	M	M	M	L	VH	L
C7	VL	M	M	H	L	H	M	M	M
C8	M	VH	VH	M	M	VL	M	VL	VH
C9	VL	H	H	H	VL	VL	M	VH	L
C10	H	M	M	H	L	VH	H	M	L
C11	H	VL	VH	VH	H	L	L	VH	VL
C12	VL	M	VH	VL	VL	VL	H	VL	M
C13	L	H	M	VH	VH	M	L	VH	VH
C14	VL	L	L	VL	VL	M	M	VH	M
C15	VH	L	H	VL	H	M	L	H	VH
C16	H	H	H	L	M	M	M	VL	M
C17	VL	H	M	VH	VL	L	L	M	H
C18	H	VL	VL	M	L	VL	L	VL	H
C19	VL	VH	M	VH	VH	M	H	VH	H

Table 5: Qualitative Assessment for the three alternatives

The qualitative ratings are converted into fuzzy triangular numbers and then aggregate ratings are generated using Eqn (2). Table 6 presents the aggregate fuzzy decision matrix for the three alternatives.

Criteria	**A1**	**A2**	**A3**	Min	Max
C1	(1,5,9)	(1,3.667,9)	(1,5.667,9)	1	9
C2	(1,1.667,5)	(1,2.333,7)	(1,3.667,9)	1	9
C3	(1,5.667,9)	(3,7.667,9)	(1,5.667,9)	1	9
C4	(1,4.333,9)	(1,2.333,7)	(1,5,9)	1	9
C5	(1,5,9)	(3,5.667,9)	(1,3,9)	1	9
C6	(1,7,9)	(3,5,7)	(1,5,9)	1	9
C7	(1,3.667,7)	(1,5.667,9)	(3,5,7)	1	9
C8	(3,7.667,9)	(1,3.667,7)	(1,5,9)	1	9
C9	(1,5,9)	(1,3,9)	(1,5.667,9)	1	9
C10	(3,5.667,9)	(1,6.333,9)	(1,5,9)	1	9
C11	(1,5.667,9)	(1,6.333,9)	(1,4.333,9)	1	9
C12	(1,5,9)	(1,1,3)	(1,4.333,9)	1	9
C13	(1,5,9)	(3,7.667,9)	(1,7,9)	1	9
C14	(1,2.333,5)	(1,2.333,7)	(3,6.333,9)	1	9
C15	(1,6.333,9)	(1,4.333,9)	(1,6.333,9)	1	9
C16	(5,7,9)	(1,4.333,7)	(1,3.667,7)	1	9
C17	(1,4.333,9)	(1,4.333,9)	(1,5,9)	1	9
C18	(1,3,9)	(1,3,7)	(1,3.667,9)	1	9
C19	(1,5,9)	(3,7.667,9)	(5,7.667,9)	1	9

Table 6. Aggregate fuzzy decision matrix for alternatives

After obtaining the fuzzy decision matrix and fuzzy/crisp criteria weights, we apply the four MCDM techniques. The results are presented as follows.

4.1 Fuzzy VIKOR

First of all, we generate aggregated crisp ratings for the three alternatives using eqn (6). Based on these values, we will compute the best f_j^* and the worst values f_j^- of the 19 criteria using eqn (7). Table 7 presents the results for the aggregated crisp ratings, f_j^* and f_j^- for the 19 criteria.

Criteria	Crisp Ratings			f_j^*	f_j^-
	A1	**A2**	**A3**		
C1	5	4.111	5.444	5.444	4.111
C2	2.111	2.889	4.111	4.111	2.111
C3	5.444	7.111	5.444	7.111	5.444
C4	4.556	2.889	5	5	2.889
C5	5	5.778	3.667	5.778	3.667
C6	6.333	5	5	6.333	5
C7	3.778	5.444	5	5.444	3.778
C8	7.111	3.778	5	7.111	3.778
C9	5	3.667	5.444	5.444	3.667
C10	5.778	5.889	5	5.889	5
C11	5.444	5.889	4.556	5.889	4.556
C12	5	1.333	4.556	5	1.333
C13	5	7.111	6.333	7.111	5
C14	2.556	2.889	6.222	6.222	2.556
C15	5.889	4.556	5.889	5.889	4.556
C16	7	4.222	3.778	7	3.778
C17	4.556	4.556	5	5	4.556
C18	3.667	3.333	4.111	4.111	3.333
C19	5	7.111	7.444	7.444	5

Table 7. The best values f_j^* and the worst values f_j^- for the 19 criteria

Table 8 presents the S_i, R_i and Q_i values for the three alternatives computed using eqn (7-9). The values of $S^*= 0.345$, $S^- = 0.603$, $R^*= 0.058$, $R^- =0.087$ are obtained using eqn (10).

	A1	**A2**	**A3**
S_i	0.506	0.603	0.345
R_i	0.069	0.087	0.058
Q_i	0.493	1	0

Table 8. S_i, R_i and Q_i values for the three alternatives

Table 9 ranks the three alternatives, sorting by the values of S_i, R_i and Q_i obtained from Table 8 in ascending order.

$\boldsymbol{S_i}$	A3	A1	A2
$\boldsymbol{R_i}$	A3	A1	A2
$\boldsymbol{Q_i}$	A3	A1	A2

Table 9: Alternative rankings

It can be seen from the results of Table 9 that alternative A3 is the best ranked by the measure Q_i(minimum). We now check it for the following two conditions (section 3.3.1).

1). C1: acceptable advantage (eqn(11)).

Using eqn (11), DQ = 1/19-1 = 1/18 = 0.055. Applying eqn (10), we find Q(A1)-Q(A3) = 0.4939-0= 0.4939 > 0.055, hence the condition $Q(A^{(2)}) - Q(A^{(1)}) \geq DQ$ is satisfied.

2). C2: Acceptable stability in decision making (eqn (12))

Since alternative A3 is also best ranked by S_i and R_i (considering the "by consensus rule $\nu \approx 0.5$"), therefore it is finally chosen and ranked the best urban transportation project.

4.2 Fuzzy TOPSIS

We begin by performing normalization of the fuzzy decision matrix of alternatives using eqns (14-15). For example, the normalized rating of alternative A1 for criteria C2 (Operating costs), which is a cost category criteria is given by:

$$a_j^- = \min_i (1,1,1) = 1$$

$$\tilde{r}_{ij} = (\frac{1}{5}, \frac{1}{1.667}, \frac{1}{1}) = (0.2, 0.6, 1)$$

The normalized value of alternative A1 for criteria C1 (Revenues) which is a benefit category criteria is given by:

$$c_j^* = \max_i (9,9,9) = 9$$

$$\tilde{r}_{ij} = (\frac{1}{9}, \frac{5}{9}, \frac{9}{9}) = (0.11, 0.556, 1)$$

Likewise, we compute the normalized values of the alternatives for the remaining criteria. The normalized fuzzy decision matrix for the 3 alternatives is presented in Table 10.

Criteria	**A1**	**A2**	**A3**
C1	(0.111,0.556,1)	(0.111,0.407,1)	(0.111,0.629,1)
C2	(0.2,0.6,1)	(0.142,0.428,1)	(0.111,0.272,1)
C3	(0.111,0.629,1)	(0.333,0.851,1)	(0.111,0.629,1)
C4	(0.111,0.481,1)	(0.111,0.259,0.778)	(0.111,0.556,1)
C5	(0.111,0.556,1)	(0.333,0.629,1)	(0.111,0.333,1)
C6	(0.111,0.778,1)	(0.333,0.556,0.778)	(0.111,0.556,1)
C7	(0.111,0.523,0.778)	(0.111,0.629,1)	(0.333,0.556,0.778)
C8	(0.333,0.851,1)	(0.111,0.407,0.778)	(0.111,0.556,1)
C9	(0.111,0.556,1)	(0.111,0.333,1)	(0.111,0.629,1)
C10	(0.111,0.176,0.333)	(0.111,0.157,1)	(0.111,0.2,1)
C11	(0.111,0.629,1)	(0.111,0.703,1)	(0.111,0.481,1)
C12	(0.111,0.556,1)	(0.111,0.111,0.333)	(0.111,0.481,1)
C13	(0.111,0.556,1)	(0.333,0.851,1)	(0.111,0.778,1)
C14	(0.111,0.259,0.556)	(0.111,0.259,0.778)	(0.333,0.703,1)
C15	(0.111,0.703,1)	(0.111,0.481,1)	(0.111,0.703,1)
C16	(0.556,1,1)	(0.111,0.481,0.778)	(0.111,0.407,0.778)
C17	(0.111,0.481,1)	(0.111,0.481,1)	(0.111,0.556,1)
C18	(0.111,0.333,1)	(0.111,0.333,0.778)	(0.111,0.407,1)
C19	(0.111,0.556,1)	(0.333,0.851,1)	(0.556,0.851,1)

Table 10: Normalized fuzzy alternative ratings

Then, the fuzzy weighted decision matrix for the three alternatives is constructed using eqn (16). The $\tilde{r}_{ij}$ values from Table 10 and fuzzy criteria weight values ($\tilde{w}_j$) from Table 4 are used to compute the fuzzy weighted decision matrix for the alternatives. For example, for alternative A1, the fuzzy weight for criteria C1 (Revenues) is given by:

$$\tilde{v}_{ij} = (0.11, 0.556, 1)(.)(1, 3, 7) = (0.11, 1.667, 7)$$

Likewise, we compute the fuzzy weights for the remaining criteria for the three alternatives. The results are presented in Table 11. Based on these values, we then calculate the fuzzy positive ideal solution (A*) and the fuzzy negative ideal solutions (A-) for each criteria using eqn (17-18). For example, for criteria C1 (Revenues), $A^{-} = (0.11, 0.11, 0.11)$ and $A^{*} = (7, 7, 7)$. The last two columns of Table 11 contain the complete results for the 19 criteria.

Criteria	**A1**	**A2**	**A3**	***A-***	***A****
C1	(0.111,1.667,7)	(0.111,1.222,7)	(0.111,1.889,7)	0.111	7
C2	(0.2,3,9)	(0.142,2.142,9)	(0.111,1.363,9)	0.111	9
C3	(0.111,2.308,9)	(0.333,3.123,9)	(0.111,2.308,9)	0.111	9
C4	(0.111,2.086,9)	(0.111,1.123,7)	(0.111,2.407,9)	0.111	9
C5	(0.111,2.778,9)	(0.333,3.148,9)	(0.111,1.666,9)	0.111	9
C6	(0.111,2.851,9)	(0.333,2.037,7)	(0.111,2.037,9)	0.111	9
C7	(0.111,2.619,7)	(0.111,3.148,9)	(0.333,2.778,7)	0.111	9
C8	(0.333,1.987,5)	(0.111,0.950,3.889)	(0.111,1.296,5)	0.111	5
C9	(0.111,1.667,7)	(0.111,1,7)	(0.111,1.889,7)	0.111	7
C10	(0.111,0.647,2.333)	(0.111,0.578,7)	(0.111,0.7333,7)	0.111	7
C11	(0.111,2.728,9)	(0.111,3.049,9)	(0.111,2.086,9)	0.111	9
C12	(0.111,2.778,9)	(0.111,0.556,3)	(0.111,2.407,9)	0.111	9
C13	(0.111,1.667,7)	(0.333,2.556,7)	(0.111,2.333,7)	0.111	7
C14	(0.111,1.641,5)	(0.111,1.641,7)	(0.333,4.456,9)	0.111	9
C15	(0.556,5.395,9)	(0.556,3.691,9)	(0.556,5.395,9)	0.556	9
C16	(0.556,3,5)	(0.111,1.444,3.889)	(0.111,1.222,3.889)	0.111	5
C17	(0.111,2.086,9)	(0.111,2.086,9)	(0.111,2.407,9)	0.111	9
C18	(0.111,1.444,9)	(0.111,1.444,7)	(0.111,1.765,9)	0.111	9
C19	(0.333,3.148,9)	(1,4.827,9)	(1.667,4.827,9)	0.333	9

Table 11: Normalized weighted fuzzy alternative ratings, FNIS and FPIS

Then, we compute the distance d_v (.) of each alternative from the fuzzy positive ideal matrix (A*) and fuzzy negative ideal matrix (A-) using eqns (21). For example, for alternative A1 and criteria C1, the distances $d_v(A_1, A^{*})$ and $d_v(A_1, A^{-})$ are computed as follows:

$$d_v(A_1, A^*) = \sqrt{\frac{1}{3}\left[(0.111-7)^2 + (1.667-7)^2 + (7-7)^2\right]} = 5.029$$

$$d_v(A_1, A^-) = \sqrt{\frac{1}{3}\left[(0.111-0.111)^2 + (1.667-0.111)^2 + (7-0.111)^2\right]} = 4.077$$

Likewise, we compute the distances for the remaining criteria for the three alternatives. The results are shown in Table 12.

Criteria	d-			d*		
	A1	**A2**	**A3**	**A1**	**A2**	**A3**
C1	4.077	4.028	4.107	5.029	5.191	4.952
C2	5.396	5.264	5.182	6.149	6.467	6.765
C3	5.286	5.420	5.286	6.423	6.045	6.423
C4	5.257	4.020	5.3004	6.501	6.953	6.389
C5	5.357	5.424	5.209	6.264	6.037	6.653
C6	5.370	4.131	5.251	6.239	6.521	6.519
C7	4.232	5.423	4.266	6.422	6.144	6.267
C8	3.026	2.234	2.904	3.206	3.72	3.541
C9	4.077	4.0102	4.107	5.029	5.274	4.952
C10	1.319	3.986	3.993	6.044	5.437	5.376
C11	5.349	5.405	5.257	6.280	6.175	6.501
C12	5.357	1.687	5.3004	6.264	7.88	6.389
C13	4.077	4.222	4.179	5.029	4.625	4.803
C14	2.957	4.074	5.713	7.051	6.761	5.649
C15	5.619	5.2007	5.619	5.301	5.758	5.301
C16	3.288	2.312	2.273	2.813	3.548	3.624
C17	5.257	5.257	5.3004	6.501	6.50	6.389
C18	5.189	4.051	5.22	6.735	6.833	6.616
C19	5.260	5.649	5.688	6.0375	5.209	4.871

Table 12: Distances of alternatives from the FNIS and FPIS.

Then, we compute the distances d_i^* and d_i^- using eqns (19) and (20) for the three alternatives. For example, for alternative A1 and criteria C1, the distances d_i^* and d_i^- are given by:

$$d_i^- = \sqrt{\frac{1}{3}\left[(0.111-0.111)^2+(1.667-0.111)^2+(7-0.111)^2\right]}+$$
$$\sqrt{\frac{1}{3}\left[(0.2-0.111)^2+(3-0.111)^2+(9-0.111)^2\right]}+..$$
$$..+\sqrt{\frac{1}{3}\left[(0.333-0.333)^2+(3.148-0.333)^2+(9-0.333)^2\right]}=85.76$$

and

$$d_i^* = \sqrt{\frac{1}{3}\left[(0.111-7)^2+(1.667-7)^2+(7-7)^2\right]}+$$
$$\sqrt{\frac{1}{3}\left[(0.2-9)^2+(3-9)^2+(9-9)^2\right]}+..$$
$$..+\sqrt{\frac{1}{3}\left[(0.333-9)^2+(3.148-9)^2+(9-9)^2\right]}=109.327$$

Then, we compute the closeness coefficient (CC_i) of the three alternatives using eqn (22). For example, for alternative A1, the closeness coefficient is given by:

$CC_i = d_i^- /(d_i^- + d_i^+)$ = 85.76/(85.76+109.327) = 0.4395

Likewise, CC_i for the other two alternatives are computed. The final results are shown in Table 13.

	A1	**A2**	**A3**
d^-	85.760	81.804	90.163
d^*	109.327	111.088	107.988
CC_i	0.4395	0.424	0.455
	A3>A1>A2		

Table 13: Closeness coefficient ratios for the three alternatives

By comparing the CC_i values of the three alternatives (Table 13), we find that A3>A1>A2. Therefore, alternative A3 is finally ranked as best and recommended for implementation.

4.3 Fuzzy SAW

Firstly, we normalize the alternative ratings for the 19 criteria using eqn (23-24).

For example, criteria C1 is of benefit type (i.e. the larger the better), therefore its normalized outcome is $= \frac{5}{\max\{5, 4.111, 5.444\}} = \frac{5}{5.444} = 0.9183$. For criteria C2, which is of cost type (i.e. the less, the better), the normalized outcome is $= \frac{\min\{2.111, 2.889, 4.111\}}{2.111} = \frac{2.111}{2.111} = 1$. Table 14 presents the normalized values for the three alternatives and the weights of the 19 criteria.

	Crisp values			Min	Max	Normalized Crisp Value		
Criteria	**A1**	**A2**	**A3**			**A1**	**A2**	**A3**
C1	5	4.111	5.444	4.111	5.444	0.918	0.755	1.000
C2	2.111	2.889	4.111	2.111	4.111	1.000	0.731	0.514
C3	5.444	7.111	5.444	5.444	7.111	0.766	1.000	0.766
C4	4.556	2.889	5	2.889	5.000	0.911	0.578	1.000
C5	5	5.778	3.667	3.667	5.778	0.865	1.000	0.635
C6	6.333	5	5	5.000	6.333	1.000	0.789	0.789
C7	3.778	5.444	5	3.778	5.444	0.694	1.000	0.918
C8	7.111	3.778	5	3.778	7.111	1.000	0.531	0.703
C9	5	3.667	5.444	3.667	5.444	0.918	0.673	1.000
C10	5.778	5.889	5	5.000	5.889	0.865	0.849	1.000
C11	5.444	5.889	4.556	4.556	5.889	0.925	1.000	0.774
C12	5	1.333	4.556	1.333	5.000	1.000	0.267	0.911
C13	5	7.111	6.333	5.000	7.111	0.703	1.000	0.891
C14	2.556	2.889	6.222	2.556	6.222	0.411	0.464	1.000
C15	5.889	4.556	5.889	4.556	5.889	1.000	0.774	1.000
C16	7	4.222	3.778	3.778	7.000	1.000	0.603	0.540
C17	4.556	4.556	5	4.556	5.000	0.911	0.911	1.000
C18	3.667	3.333	4.111	3.333	4.111	0.892	0.811	1.000
C19	5	7.111	7.444	5.000	7.444	0.672	0.955	1.000

Table 14: Crisp alternative ratings

Using eq (25), we now calculate the overall performance rating for the three alternatives. For example, for alternative A1, the overall performance rating $= 0.918 * 3.33 + 1 * 5 + ... + 0.672 * 5.778 = 72.666$. Table 15 presents the overall performance ratings for the three alternatives. It can be seen that alternative A3 has the highest value, therefore it is finally selected.

Criteria	Normalized crisp value			Criteria Weight	Weighted criteria value		
	A1	A2	A3		A1	A2	A3
C1	0.918	0.755	1.000	3.333	3.061	2.517	3.333
C2	1.000	0.731	0.514	5.000	5.000	3.654	2.568
C3	0.766	1.000	0.766	4.111	3.148	4.111	3.148
C4	0.911	0.578	1.000	4.556	4.151	2.632	4.556
C5	0.865	1.000	0.635	5.000	4.327	5.000	3.173
C6	1.000	0.789	0.789	4.111	4.111	3.246	3.246
C7	0.694	1.000	0.918	5.000	3.469	5.000	4.592
C8	1.000	0.531	0.703	2.556	2.556	1.358	1.797
C9	0.918	0.673	1.000	3.333	3.061	2.245	3.333
C10	0.865	0.849	1.000	3.778	3.269	3.208	3.778
C11	0.925	1.000	0.774	4.556	4.212	4.556	3.524
C12	1.000	0.267	0.911	5.000	5.000	1.333	4.556
C13	0.703	1.000	0.891	3.333	2.344	3.333	2.969
C14	0.411	0.464	1.000	5.889	2.419	2.734	5.889
C15	1.000	0.774	1.000	7.444	7.444	5.759	7.444
C16	1.000	0.603	0.540	3.000	3.000	1.810	1.619
C17	0.911	0.911	1.000	4.556	4.151	4.151	4.556
C18	0.892	0.811	1.000	4.556	4.063	3.694	4.556
C19	0.672	0.955	1.000	5.778	3.881	5.519	5.778
				Total	**72.666**	**65.858**	**74.412**
					A3>A1>A2		

Table 15: Crisp Normalized alternative ratings

4.4 Fuzzy GRA

Firstly, we normalize the alternative data using eqn (26-28). Since C1 is a benefit type criterion, it is normalized using eqn (26) as follows.

$$\tilde{r}_{ij} = \frac{5 - 4.111}{5.444 - 4.111} = 0.667$$

The normalized value for criteria C2 (cost criteria) is obtained using eqn (27):

$$= \frac{4.111 - 2.111}{4.111 - 2.111} = 1 \text{(The least the better type criteria)}$$

There is no the closer to the goal type criteria (eqn (28)) in our study. Table 16 presents the normalized values of the 19 criteria for the three alternatives. The reference sequence for the normalized criteria $X_0 = (x_{01}, x_{02}, ..., x_{019})$ containing the ideal solutions and is presented in the last column of Table 16. It can be seen that for benefit type criteria, it is the maximum value and for cost type criteria it is the minimum value.

Criteria	Crisp			Min	Max	Normalized value			Reference sequence
	A1	**A2**	**A3**			**A1**	**A2**	**A3**	
C1	5	4.111	5.444	4.111	5.444	0.667	0	1	1
C2	2.111	2.889	4.111	2.111	4.111	1	0.611	0	0
C3	5.444	7.111	5.444	5.444	7.111	0	1	0	1
C4	4.556	2.889	5	2.889	5	0.789	0	1	1
C5	5	5.778	3.667	3.667	5.778	0.631	1	0	1
C6	6.333	5	5	5	6.333	1	0	0	1
C7	3.778	5.444	5	3.778	5.444	0	1	0.733	1
C8	7.111	3.778	5	3.778	7.111	1	0	0.367	1
C9	5	3.667	5.444	3.667	5.444	0.75	0	1	1
C10	5.778	5.889	5	5	5.889	0.125	0	1	0
C11	5.444	5.889	4.556	4.556	5.889	0.667	1	0	1
C12	5	1.333	4.556	1.333	5	1	0	0.878	1
C13	5	7.111	6.333	5	7.111	0	1	0.631	1
C14	2.556	2.889	6.222	2.556	6.222	0	0.090	1	1
C15	5.889	4.556	5.889	4.556	5.889	1	0	1	1
C16	7	4.222	3.778	3.778	7	1	0.137	0	1
C17	4.556	4.556	5	4.556	5	0	0	1	1
C18	3.667	3.333	4.111	3.333	4.111	0.428	0	1	1
C19	5	7.111	7.444	5	7.444	0	0.863	1	1

Table 16: Normalized alternative ratings and Reference Sequence

Now, we calculate the grey relational coefficient $\gamma(x_{0j}, x_{ij})$ for the three alternatives using eqn (29). For example, for alternative A1, criteria C1,

ζ =0.5

$$\Delta_{11} = |0.667 - 1| = 0.333$$
$$\Delta_{\min} = Min\{\Delta_{11}, \Delta_{12}, ..., \Delta_{193}\} = Min\{0.333, 1, 0, ..., 0\} = 0$$
$$\Delta_{\max} = Max\{\Delta_{11}, \Delta_{12}, ..., \Delta_{193}\} = Max\{0.333, 1, 0, ..., 0\} = 1$$
$$\gamma(x_{01}, x_{11}) = \frac{0 + 0.5*1}{0.333 + 0.5*1} = 0.6$$

Table 17 presents the results of grey relational coefficient. The last column presents the normalized criteria weights. The distinguishing coefficient is set equal to 0.5.

Criteria	Delta (Δ)			Grey coefficient (γ)			Criteria weight (crisp)
	A1	**A2**	**A3**	**A1**	**A2**	**A3**	
C1	0.333	1	0	0.6	0.333	1	0.039
C2	1	0.611	0	1	0.562	0.333	0.0589
C3	1	0	1	0.333	1	0.333	0.0484
C4	0.210	1	0	0.703	0.333	1	0.0536
C5	0.368	0	1	0.575	1	0.333	0.0589
C6	0	1	1	1	0.333	0.333	0.0484
C7	1	0	0.266	0.333	1	0.652	0.0589
C8	0	1	0.633	1	0.333	0.441	0.0301
C9	0.25	1	0	0.667	0.333	1	0.0392
C10	0.125	0	1	0.363	0.333	1	0.0445
C11	0.333	0	1	0.6	1	0.333	0.0536
C12	0	1	0.121	1	0.333	0.804	0.0589
C13	1	0	0.368	0.333	1	0.575	0.0392
C14	1	0.909	0	0.333	0.354	1	0.0693
C15	0	1	0	1	0.333	1	0.0876
C16	0	0.862	1	1	0.367	0.333	0.0353
C17	1	1	0	0.333	0.333	1	0.0536
C18	0.571	1	0	0.467	0.333	1	0.0536
C19	1	0.136	0	0.333	0.785	1	0.0680
delta min		0					
delta max		1					

Table 17: Grey relational coefficients

Now, we calculate the grey relational grade for the three alternatives using eqn (29). For example, for alternative A1,

$$\Gamma(X_0, X_1) = 0.6*0.039 + 0.1*0.0589 + ... + 0.333*0.068 = 0.6267$$

Table 18 presents the results for the grey relational grade for the three alternatives. It can be seen that alternative A3 has the highest value and therefore it is finally chosen.

Criteria	Weighted grey coefficient (Γ)		
	A1	A2	A3
C1	0.023	0.013	0.039
C2	0.058	0.033	0.019
C3	0.016	0.048	0.016
C4	0.037	0.017	0.053
C5	0.033	0.058	0.019
C6	0.048	0.016	0.016
C7	0.019	0.058	0.038
C8	0.030	0.010	0.013
C9	0.026	0.013	0.039
C10	0.016	0.014	0.044
C11	0.032	0.053	0.017
C12	0.058	0.019	0.047
C13	0.013	0.039	0.022
C14	0.023	0.024	0.069
C15	0.087	0.029	0.087
C16	0.035	0.012	0.011
C17	0.017	0.017	0.053
C18	0.025	0.017	0.053
C19	0.022	0.053	0.068
Grey Relational grade	**0.626**	**0.553**	**0.732**
A3>A1>A2			

Table 18: Grey relational grade

Table 19 presents the ranking results obtained from the application of four MCDM techniques. '1' represents the highest rank and '3' represents the lowest rank. It can be seen from Table 19 that the results of the four techniques are in agreement with each other as the alternative A3 is rated best (sustainable) by all the four techniques. Therefore, using the veto technique (or majority winner rule), alternative A3 is finally chosen as the best urban mobility project from sustainability perspective and is recommended for implementation.

Technique	**A1**	**A2**	**A3**
Fuzzy TOPSIS	2	3	1
Fuzzy VIKOR	2	3	1
Fuzzy SAW	2	3	1
Fuzzy GRA	2	3	1
	A3>A1>A2		

Table 19: Final Ranking Results

Please note that the above results are based on artificial numbers and used for demonstration purposes only. Interested readers (or practitioners) can use real data and apply the above four techniques in a similar manner to seek accurate results.

5. CONCLUSIONS AND FUTURE WORK

In this paper, we perform multicriteria decision making (MCDM) for sustainability evaluation of urban mobility projects using qualitative data. A three step approach is proposed. In the first step, we perform selection of evaluation criteria using literature review. In the second step, we generate criteria and alternative ratings using expert opinions. Qualitative data is used for rating the criteria and the alternatives which are then transformed into fuzzy triangular numbers for further processing through multicriteria decision making techniques in step 3. The multicriteria decision making techniques used in our study are Fuzzy TOPSIS, Fuzzy VIKOR, Fuzzy SAW and Fuzzy GRA.

In this paper, we have limited our study to identification of sustainability evaluation criteria and demonstrating the application of four comparabale MCDM techniques for urban mobility project selection. In our future works, we intend to perform a).more numerical experiments to determine if these results always remain consistent irrespective of input data and b).assess the influence of criteria weights on final results using sensitivity analysis.

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